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ROTOR-BEARING DYNAMICS TECHNOLOGY DESIGN GUIDE
Part III
Tapered Roller Bearings

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February 1979

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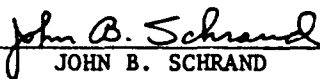
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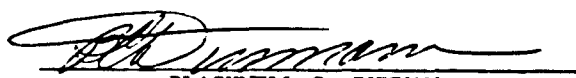

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report is an update of the original Part IV of the Rotor-Bearing Dynamics Design Technology Series, AFAPL-TR-65-45 (Parts I through X). A computer program is given for preparation of tapered roller bearing stiffness data input for rotordynamic response programs. The complete stiffness matrix is calculated including centrifugal effects. Considerations such as elastohydrodynamic and cage effects are not included since they have little influence on the calculation of tapered roller bearing stiffness. The resulting program is reasonably small and easy to use.		

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FOREWORD

This report was prepared by Shaker Research Corporation under USAF Contract No. AF33615-76-C-2038. The contract was initiated under Project 3048, "Fuels, Lubrication, and Fire Protection," Task 304806, "Aerospace Lubrication," Work Unit 30480685, "Rotor-Bearing Dynamics Design."

The work reported herein was performed during the period 15 April 1977 to 15 November 1978, under the direction of John B. Schrand (AFAPL/SFL) and Dr. James F. Dill (AFAPL/SFL), Project Engineers. The report was released by the authors in December 1978.

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NOMENCLATURE

<u>Symbol</u>	<u>Description</u>	<u>Units</u>
b_x	Semi-width of contact ellipse at x	in.
B_1	Corner break at roller small end	in.
B_2	Corner break at roller big end	in.
B_{ij}	Damping component, change of force in i direction due to velocity in j direction; i = x, y, z; j = x, y, z.	$\frac{\text{lb-sec}}{\text{in}}$
\underline{B}_N	Damping matrix $\begin{bmatrix} (\underline{B}_N)_{\text{lineal}} & 0 & 0 \\ & 0 & 0 \\ 0 & 0 & (\underline{B}_N)_{\text{angular}} \\ 0 & 0 & \end{bmatrix}$	$\frac{\text{lb-sec}}{\text{in}}$
$(\underline{B})_{\text{lineal}}$	Damping matrix due to lateral velocities $\begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{\text{lineal}}$	$\frac{\text{lb-sec}}{\text{in}}$
$(\underline{B})_{\text{angular}}$	Damping matrix due to angular velocities $\begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{\text{angular}}$	$\frac{\text{in-lb-sec}}{\text{radian}}$
C_i	A constant, $C_i = \begin{cases} 1 & \text{for } i = 1 \\ -1 & \text{for } i = 2 \end{cases}$	-
d	Roller diameter at midpoint of effective length of roller	in.
d_x	Roller diameter at x	in.
E	Pitch diameter at midpoint of effective length	in.
E_E	Modulus of elasticity for roller	lbs/in^2

E_R	Modulus of elasticity for race body	lbs/in ²
E_x	Pitch diameter at x	in.
F_c	Roller centrifugal force	lbs.
F_i	External applied force, $i = x, y, z$	lbs.
F'_i	Reaction force, positive in direction opposite to displacements, $i = x, y, z$	lbs.
\underline{F}	Force Matrix = $\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$	lbs.
G	Distance along roller cone element from extreme end of effective length to point where crown drop is measured	in.
H	Roller crown radius minus the rise of the arc at midpoint of effective length	in.
I_{cg}	Moment of inertia about roller center of gravity	lbs-in ²
K	Roller-race stiffness	lbs/in.
K_{ij}	Stiffness component, change of force in i direction due to displacement in j direction. $i = x, y, z; j = x, y, z$	lbs/in.
\underline{K}_N	Stiffness matrix $\begin{bmatrix} (K_N)_{\text{lineal}} & 0 & 0 \\ & 0 & 0 \\ 0 & 0 & (K_N)_{\text{angular}} \\ 0 & 0 & \end{bmatrix}$	
$(\underline{K})_{\text{lineal}}$	Stiffness matrix due to lateral displacements $\begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}_{\text{lineal}}$	lbs/in.
$(\underline{K})_{\text{angular}}$	Stiffness matrix due to angular rotations $\begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}_{\text{angular}}$	$\frac{\text{in-lb}}{\text{rad}}$

l	Perpendicular distance from the line of action of flange reaction, P_3 , to roller centerline at midpoint of effective length	in.
l_e	Effective length of roller load carrying surface	in.
l_F	Length of flat portion of roller measured along roller cone element	in.
l_T	Total length of roller measured parallel to roller axis between sharp intersections of end faces with roller cone elements	in.
m	Mass of roller	lbs.
M_G	Roller gyroscopic moment	lbs-in.
M_i	External applied moment, $i = x, y, z$	lbs-in.
M'_i	Reaction moment, $i = x, y, z$	lbs-in.
M_1	Outer race/roller contact moment	lbs-in.
M_2	Inner race/roller contact moment	lbs-in.
n	Number of rollers	
N_1	Outer ring rotational speed	rad/sec.
N_2	Inner ring rotational speed	rad/sec.
P_x	Contact unit loading	lbs/in.
P'_x	Current estimate of contact unit loading	lbs/in.
P_D	Diametral clearance	in.
P_{1q}	Outer contact load on qth roller	lbs.
P_{2q}	Inner contact load on qth roller	lbs.
P_{3q}	Flange reaction on qth roller	lbs.
q	Roller position index	-
R_c	Roller crown radius	in.
R_E	Roller big-end spherical radius	in.
V	Radial distance from roller center line to flange reaction	in.

\underline{W}_N	Column Matrix = $\begin{bmatrix} \delta_x \\ \delta_y \\ \theta_x \\ \theta_y \end{bmatrix}$	
x, y, z	Bearing coordinate system	in.
x_o	Static component of displacement	in.
x'	Dynamic component of displacement	in.
X_A	Big-end extremity of contact pattern measured parallel to roll axis from the midpoint of the effective length	
X_B	Small end extremity of contact pattern measured parallel to roller axis from the midpoint of the effective length	in.
X_A^*	Maximum permissible distance of big-end pattern extremity from midpoint of effective length measured along race	in.
X_B^*	Maximum permissible distance of small-end pattern extremity from midpoint of effective length measured along race	in.
\underline{Z}_N	Impedance matrix = $\underline{K}_N + i \underline{v}_N$	

Other notations as defined in text.

GREEK SYMBOLS

α	Angle between roller axis and line of action of flange reaction	radians
β	Outer ring contact angle	radians
γ_1	$d_x \cos \beta / E_x$	-
γ_2	$d_x \cos(\beta - \tau) / E_x$	-
δ	Displacement	in.
δ_x	Linear displacement in x direction	in.
δ_y	Linear displacement in y direction	in.
δ_z	Linear displacement in z direction	in.
Δ	Approach of inner race to outer race at midpoint of effective length	in.
Δ_x	Approach of roller to race at x	in.
Δ_{1q}	Approach of roller to outer race (cup) at qth roller	in.
Δ_{2q}	Approach of roller to inner race (cone) at qth roller	in.
ϵ_i	Residues of simultaneous equations	-
η_E	Roller elastic constant = $\frac{4(1 - \nu_E^2)}{E_E}$	$\frac{\text{in}^2}{\text{lb.}}$
η_R	Race elastic constant = $\frac{4(1 - \nu_R^2)}{E_R}$	
θ_x	Angular rotation about x axis	radians, °
θ_y	Angular rotation about y axis	radians, °
θ_z	Angular rotation about z axis	radians, °
ν	Frequency of rotation	rad/sec.
ν_E	Poisson's ratio for roller	

ν_R	Poisson's ratio for race	
	Material density	lbs/in ³
τ	Included roller cone angle	radians, ^o
ϕ	Circumferential roller position	radians, ^o
ω_R	Angular velocity of roller about its own center	rad/sec.
Ω	Orbital velocity of roller	rad/sec.
∇	Crown drop	in.

SUBSCRIPTS

<u>Symbol</u>	<u>Description</u>
b	Refers to bearing
cg	Refers to center of gravity
E	Refers to roller
F	Refers to flat
g	Refers to gyroscopic
i	Index, $i = 1, 2, 3$ or $i = x, y, z$
i,j	Refers to index of stiffness matrix; i.e., force in i direction due to displacement in j direction
p	Refers to pedestal
q	Refers to roller circumferential position
R	Refers to roller
T	Refers to total
x	Refers to x direction
y	Refers to y direction
z	Refers to z direction
1	Refers to outer race
2	Refers to inner race

SECTION I

INTRODUCTION

The original Rotor-Bearing Dynamics Design Technology Series AFAPL-TR-65-45 (Parts I through X) included a volume, Part IV(1), which presented design data for typical deep-groove and angular contact ball bearings. The data was presented in graphical form and consisted of direct radial stiffness, load carrying capacity, and load levels. In addition design guidelines and limitations were discussed. The major deficiencies of this original volume were that centrifugal effects due to high speed were ignored, and axial and angular stiffness information were omitted.

Subsequent to the publication of Part IV, several extensive treatments of rolling element bearings including elastohydrodynamic, thermal, and cage effects have been published. The computer program of Mauriello, LaGasse, and Jones (2) considers both elastohydrodynamic and cage effects for ball bearings. The more recent computer based design guide prepared by Crecelius and Pirvics (3) treats elastohydrodynamic, thermal, and cage effects for a system of ball and roller bearings.

Thus, very sophisticated analytical tools are available for the design and application of rolling element bearings. Neither of these tools, however, provide the user with the stiffness matrix required for solution of rotor dynamics problems. In addition both computer programs are very large and require an extensive computer facility for use.

Part II(4) of the revised series provided an update of the original Part IV(1). Those aspects of the original Part IV(1) which treated general design aspects of ball bearings, load capacity, speed limitations, etc. were deleted since their coverage is superficial compared to the more sophisticated computer tools now available (2,3). Only those parts directly connected with preparation of input for the rotordynamic response

programs (Part I(5) of the revised series) were retained. The stiffness data included in the original Part IV were also updated.

The present volume (Part III of the revised series) extends the treatment of rolling element bearings to the tapered roller bearing. The complete stiffness matrix is calculated including centrifugal effects. Considerations such as elastohydrodynamic and cage effects are not included since they have little influence on the calculation of tapered roller bearing stiffness. The resulting program (Appendix) is reasonably small and easy to use.

SECTION II

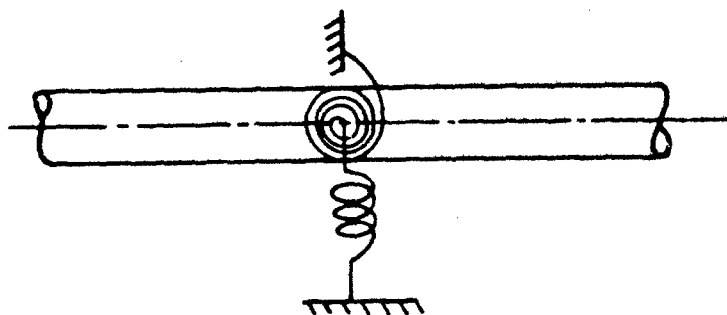
ANALYSIS

2.1 General Bearing Model and Coordinate System

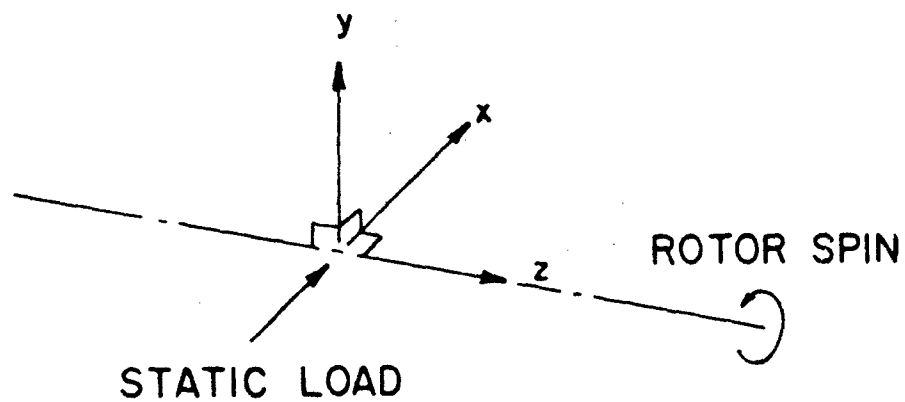
Accurate calculation of the lateral dynamic response of a high-speed rotor depends on realistic characterization of the support bearings. In the most general case, both linear and angular motions are restrained by the support bearings at the attachment location. In the analytical model, the reaction force and the reaction moment of each bearing are felt by the rotor through a single station of the rotor axis. As schematically illustrated in Figure 1a, a coil spring restraining the lateral displacement and a torsion spring which tends to oppose an inclination are attached to the same point of the rotor axis. A complete description of the characteristics of the support bearings, however, involves much more than the specification of the two spring constants. This is because:

- . The lateral motion of the rotor axis is concerned with two displacement components and two inclination components.
- . The restraining characteristics may include cross coupling among various displacement/inclination coordinates.
- . The restraining force/moment may not be temporally in phase with the displacement/inclination.
- . The restraining characteristics of the bearing may be dependent on either the rotor speed or the frequency of vibration, or both.
- . Bearing pedestal compliance may not be negligible.

To accommodate the above considerations, the support bearing characteristics are described in Reference 5 by a four-degrees-of-freedom impedance matrix as defined in Equation (1):



(FIG 1a) Bearing Stiffness Model



(FIG 1b) Bearing Location Coordinate System

$$\underline{R}_N = - \underline{Z}_N \cdot \underline{W}_N \quad (1)$$

where \underline{W}_N is a column vector containing elements which are the two lateral displacements (δ_x, δ_y) and the two lateral inclinations (θ_x, θ_y) of the rotor axis at the bearing station N.

Employing a right-handed Cartesian representation in a lateral plane as depicted in Figure 1b, the z-axis is coincident with the spin vector of the rotor. The x-axis is oriented in the direction of the external static load, and the y-axis is perpendicular to both z and x axes forming the right-handed triad (x, y, z). (δ_x, δ_y) are respectively lateral lineal displacement components of the rotor axis along the (x, y) directions. (θ_x, θ_y) are lateral inclination components respectively in the (z-x, z-y) planes. Note that θ_y is a rotation about the y-axis, while θ_x is a rotation about the negative x-axis.

\underline{Z}_N is a complex (4 x 4 matrix), and in accordance with the common notation for stiffness and damping coefficients, may be expressed as

$$\underline{Z}_N = \underline{K}_N + i\nu \underline{B}_N \quad (2)$$

where \underline{K}_N is the stiffness matrix and \underline{B}_N is the damping matrix. ν is the frequency of vibration. Most commonly, lateral lineal and angular displacements do not interact with each other so that the non-vanishing portions of \underline{K}_N and \underline{B}_N are separate 2 x 2 matrices. That is

$$\underline{K}_N = \begin{bmatrix} (\underline{K}_N)_{\text{lineal}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (\underline{K}_N)_{\text{angular}} \end{bmatrix} \quad (3)$$

$$\underline{\underline{B}}_N = \begin{bmatrix} (\underline{\underline{B}}_N)_{\text{lineal}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (\underline{\underline{B}}_N)_{\text{angular}} \end{bmatrix} \quad (4)$$

Accordingly, a total characterization of a support bearing would include sixteen coefficients which make up the 4 (2 x 2) matrices:

$$(\underline{\underline{K}})_{\text{lineal}} = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}_{\text{lineal}} \quad (5)$$

$$(\underline{\underline{B}})_{\text{lineal}} = \begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{\text{lineal}} \quad (6)$$

$$(\underline{\underline{K}})_{\text{angular}} = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}_{\text{angular}} \quad (7)$$

$$(\underline{\underline{B}})_{\text{angular}} = \begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{\text{angular}} \quad (8)$$

In the event that the pedestal compliance is significant, then the effective support impedance can be calculated from

$$\underline{\underline{Z}}_N = (\underline{\underline{Z}}_b^{-1} + \underline{\underline{Z}}_p^{-1}) \quad (9)$$

where subscripts "p" and "b" refer to the pedestal and bearing respectively. Note that both pedestal inertia and damping may be included in $\underline{\underline{Z}}_p$.

2.2 General Bearing Support Characteristics

The function of a bearing is to restrict the rotor axis to a nominal axis under realistic static and dynamic load environments. Deviation of any particular point of the rotor axis from the nominal line can be characterized by three lineal and two angular displacements. These may be designated as $(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)$ in accordance with a right-handed Cartesian reference system. The z-coordinate is coincident with the reference axis and is directed toward the spin vector. (θ_x, θ_y) are rotor axis inclinations respectively in the z-x and z-y planes. The x-coordinate is directed toward the predominant static load; e.g., earth gravity. Ideally, the bearing would resist the occurrence of any displacement so that the reaction force system imparted by the bearing to the rotor is generally expressed in matrix notation as

$$\underline{F} = \underline{Z} \cdot \underline{x} \quad (10)$$

\underline{F} is a column vector comprising the five reaction components $(F_x, F_y, F_z, M_x, M_y)$, while \underline{x} is the displacement vector $(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)$. \underline{Z} is a (5×5) matrix containing the elements Z_{ij} with both indices (i, j) ranging from 1 to 5. The values of Z_{ij} characterize how rotor displacements are being resisted by the bearing.

From the standpoint of dynamic perturbation, distinction is made between a static equilibrium component and a dynamic perturbation component for both the displacements and the reactions. Thus,

$$\underline{x} = \underline{x}_0 + \underline{x}'; \quad \underline{F} = \underline{F}_0 + \underline{F}' \quad (11)$$

$(\underline{x}', \underline{F}')$ are respectively presumed to be infinitesimal in comparison with $(\underline{x}_0, \underline{F}_0)$. Accordingly, Z_{ij} are regarded as dependent on \underline{x}_0 but not on \underline{x}' . To illustrate the idea of perturbation linearization, one may examine the one-dimensional load-displacement curve shown in Figure 2.

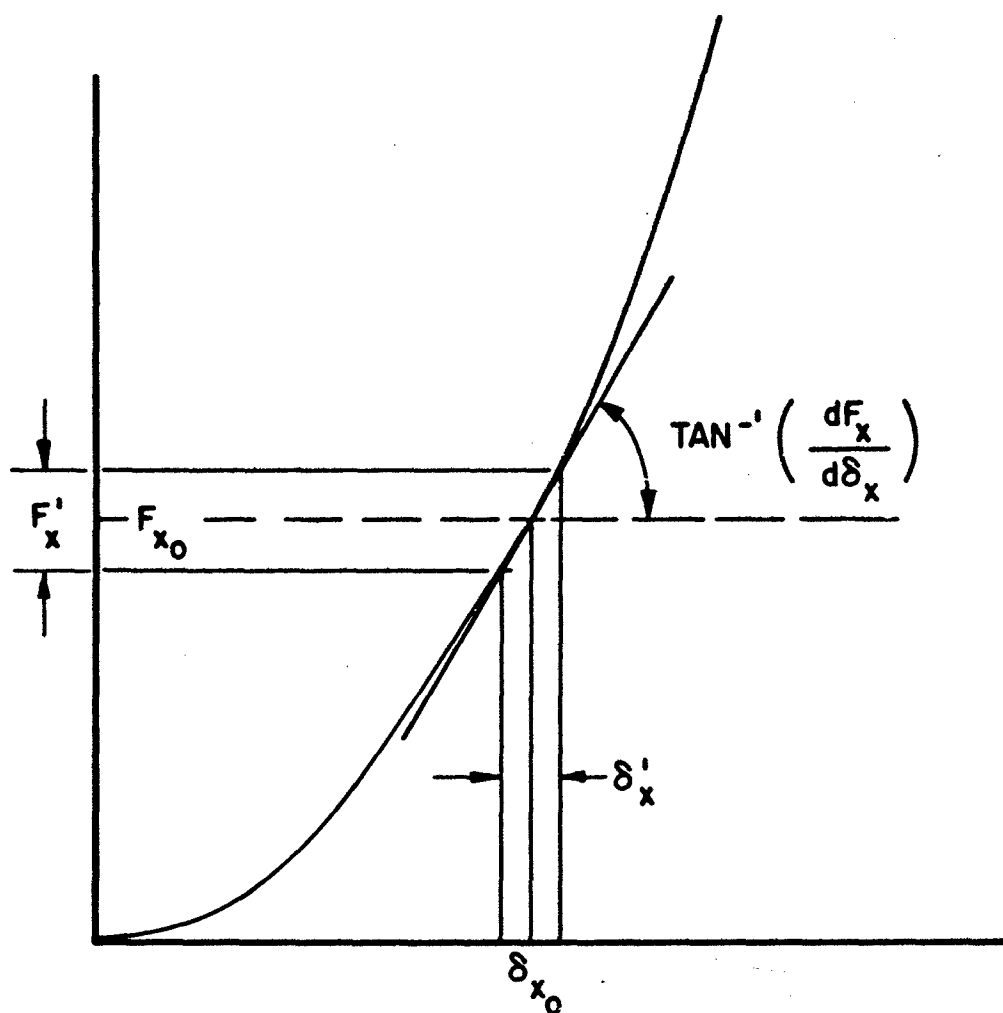


Figure 2. Linearization of Tapered Roller Bearing Stiffness

As illustrated, the load-displacement relationship is a 10/9 power law in accordance with the Hertzian point contact formula. It is not possible to describe the entire range by a linear approximation. However, if a small dynamic perturbation is taken around a static equilibrium point, $\delta'_x < \delta_{x_0}$, the small segment of the load-displacement curve can be approximated by a local tangent line. The corresponding force increment is

$$F'_x = \frac{\partial F_x}{\partial \delta_x} \delta'_x \quad (12)$$

where δ'_x is the incremental displacement. $\partial F_x / \partial \delta_x$ will depend on the amplitude of δ_{x_0} .

The question of history dependence is resolved by regarding \underline{x}' as periodic motions at any frequency ν of interest, and Z_{ij} accordingly would have both real and imaginary parts and may also be dependent on both the rotor speed ω and the vibration frequency ν .

To avoid notational clumsiness, the primes will be dropped from $(\underline{F}', \underline{x}')$ which are understood to be dynamic perturbation quantities unless the subscript "0" is used to designate the static equilibrium condition.

2.3 Tapered Roller Bearing Characterization

In many ways the tapered roller bearing is much simpler to model from a rotor dynamic point of view than a fluid film bearing. In general, the following two simplifications can be made:

- . The restraining characteristics do not include cross coupling among the various displacement/inclination coordinates.
- . The restraining force/moment is normally temporally in phase with the displacement/inclination.

Figure 3 shows a tapered roller bearing referred to in an orthogonal xyz coordinate system. The outer ring is fixed but the inner ring may move with respect to the coordinate system. Both rings are free to rotate about their axes.

Three lineal displacements, δ_x , δ_y , δ_z , and two angular displacements, θ_x , θ_y , are required to define the spatial position and attitude of the inner ring when it is displaced from its initial position. For purposes of derivation the initial situation is that existing when the bearing's end play is just taken up in the thrust direction. Figure 4 shows these displacements in the positive sense. Figure 5 establishes the convention of the roller-position index q.

2.3.1 Stiffness

The total characterization of a tapered roller bearing's stiffness can be expressed by the matrix.

$$[K] = \begin{bmatrix} \frac{\partial F_x}{\partial x} & \frac{\partial F_x}{\partial y} & \frac{\partial F_x}{\partial z} & \frac{\partial F_x}{\partial \theta_x} & \frac{\partial F_x}{\partial \theta_y} \\ \frac{\partial F_y}{\partial x} & \frac{\partial F_y}{\partial y} & \frac{\partial F_y}{\partial z} & \frac{\partial F_y}{\partial \theta_x} & \frac{\partial F_y}{\partial \theta_y} \\ \frac{\partial F_z}{\partial x} & \frac{\partial F_z}{\partial y} & \frac{\partial F_z}{\partial z} & \frac{\partial F_z}{\partial \theta_x} & \frac{\partial F_z}{\partial \theta_y} \\ \frac{\partial M_x}{\partial x} & \frac{\partial M_x}{\partial y} & \frac{\partial M_x}{\partial z} & \frac{\partial M_x}{\partial \theta_x} & \frac{\partial M_x}{\partial \theta_y} \\ \frac{\partial M_y}{\partial x} & \frac{\partial M_y}{\partial y} & \frac{\partial M_y}{\partial z} & \frac{\partial M_y}{\partial \theta_x} & \frac{\partial M_y}{\partial \theta_y} \end{bmatrix} \quad (13)$$

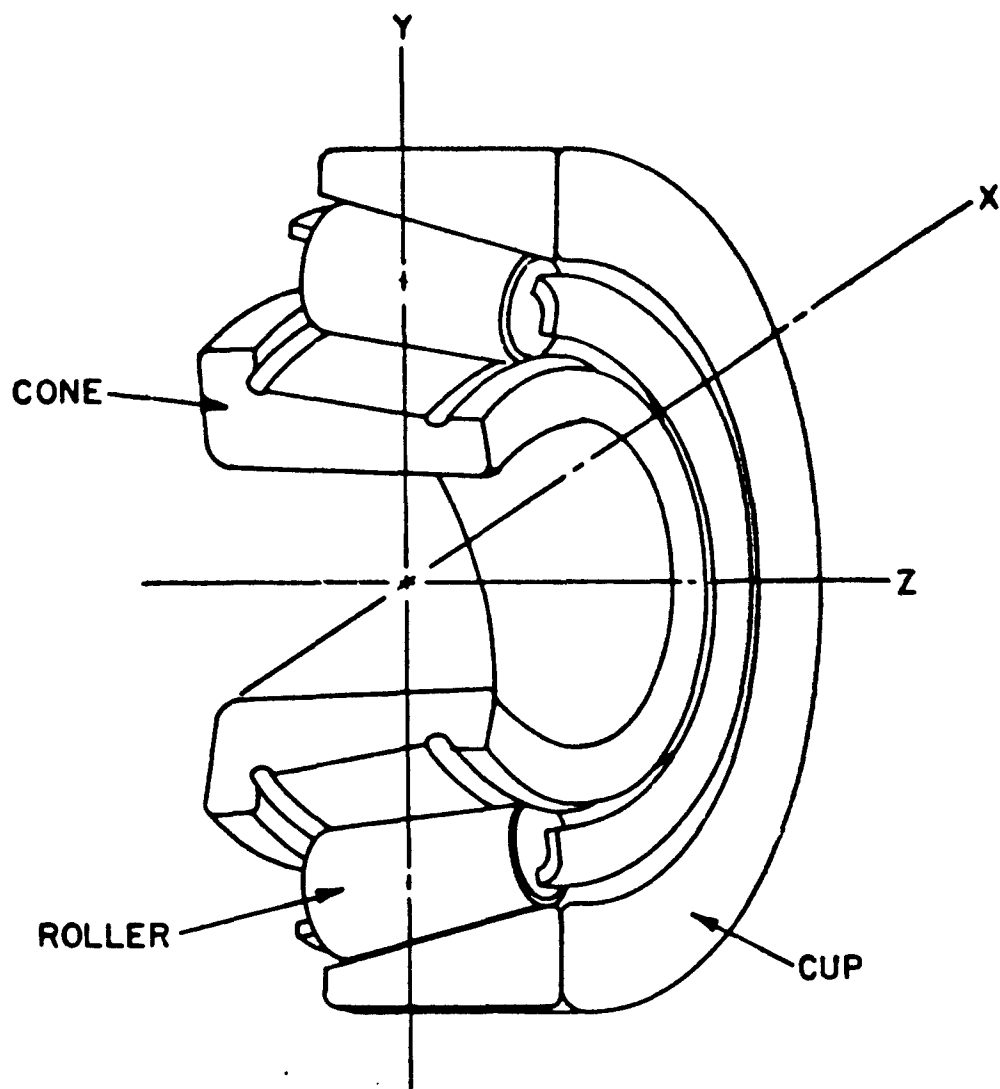


Figure 3. Tapered Roller Bearing

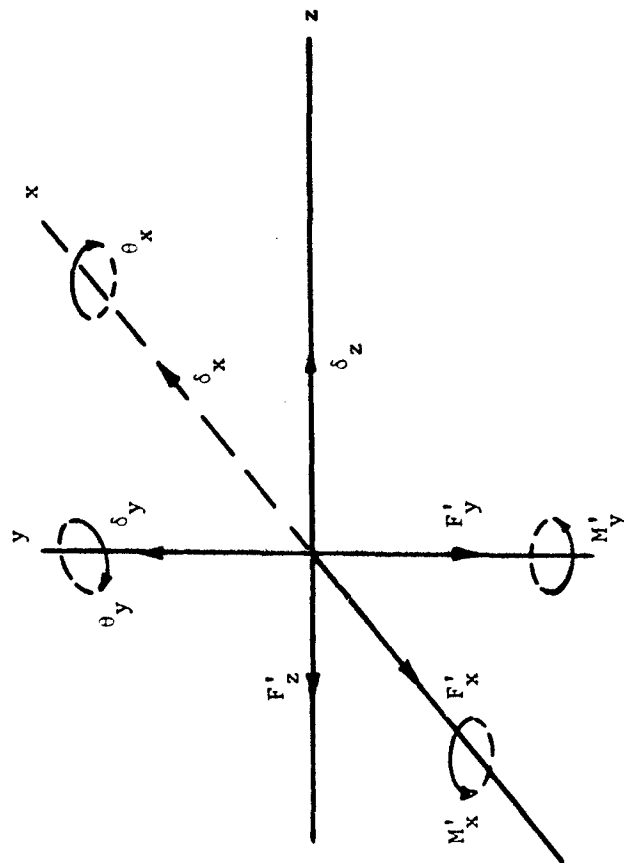


Figure 4. Bearing Coordinate System

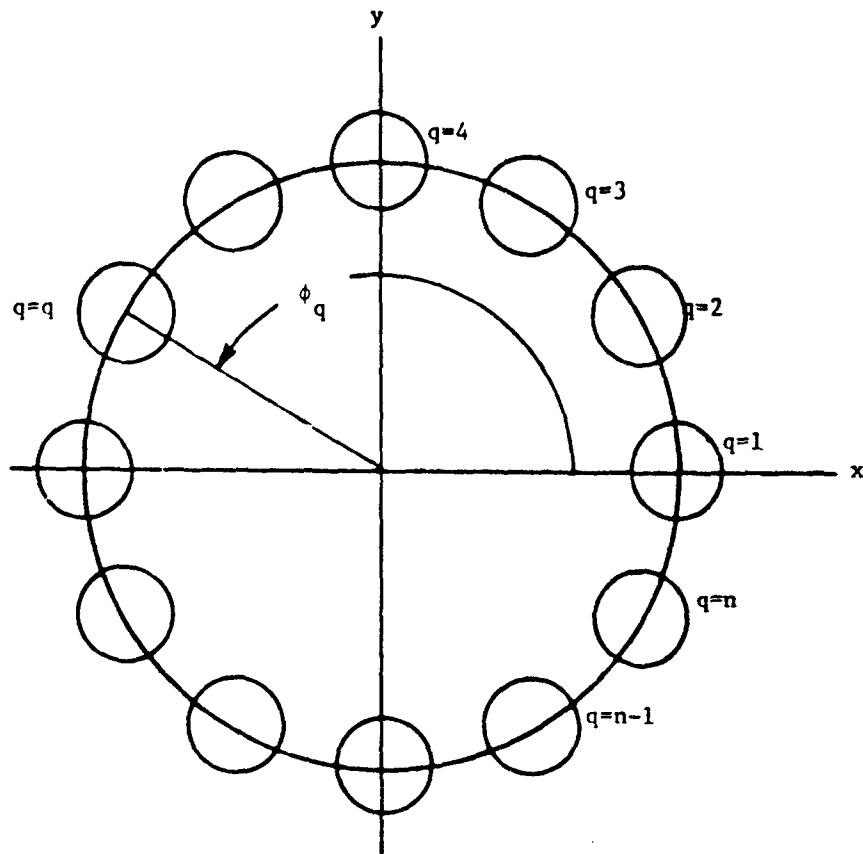


Figure 5. Tapered Roller Bearing Index, q

The lineal and angular stiffness matrices (Equations 5 and 7) can be derived from Equation (13). For example:

$$(\underline{K})_{\text{lineal}} = \begin{bmatrix} \frac{\partial F_x}{\partial x} & \frac{\partial F_x}{\partial y} \\ \frac{\partial F_y}{\partial x} & \frac{\partial F_y}{\partial y} \end{bmatrix} \quad (14)$$

$$(\underline{K})_{\text{angular}} = \begin{bmatrix} \frac{\partial M_x}{\partial \theta_x} & \frac{\partial M_x}{\partial \theta_y} \\ \frac{\partial M_y}{\partial \theta_x} & \frac{\partial M_y}{\partial \theta_y} \end{bmatrix} \quad (15)$$

Note that although the axial components of stiffness are not utilized by the lateral rotor dynamics program (5), they have been retained in the general tapered roller bearing stiffness matrix, Equation (13). The axial stiffness would be required, for example, if the reader was calculating the axial natural frequency of a tapered roller bearing mounted shaft.

2.3.2 Damping

An extensive search of the literature revealed no experimental damping data for tapered roller bearings. As the current state-of-the-art does not permit accurate calculation of tapered roller bearing damping, no damping data is included in this report.

2.4 Tapered Roller Bearing Under Combined Loading

Solution for the stiffness matrix of a tapered roller bearing under combined loading is a tedious problem and requires the use of a digital computer. In this section, the derivation of the solution is described. A computer program for obtaining the solution is included in the Appendix.

2.4.1 Bearing Applied Forces and Moments

As the result of the five displacements described previously in Figures 3 and 4, there are the reactions F'_x , F'_y , F'_z , and M'_x and M'_y . F'_x , F'_y , and F'_z are forces. M'_x and M'_y are moments. All are shown in their positive sense in Figure 4. External forces F_x and F_z may be applied at the inner ring center. The senses of the signs are the same as for the reactions F'_x and F'_z .

2.4.2 Roller Geometry

Figure 6 shows the boundary dimensions of a typical tapered roller. Roller mass, moment of inertia, and location of the center of gravity are calculated assuming the roller is a flat-ended, truncated cone bounded by R_1 , R_2 , and l_t .

In general, the big-end face of the roller is not flat but is a sphere having the radius R_e which is generally a proportion of the slant height, l_s , of the untruncated roller cone. Roller crown and corner breaks are also omitted from mass and moment of inertia calculations as their contributions are second order.

Figure 7 is a more complete sketch of the roller showing the details of the roller crown. The big-end spherical surface is neglected here also.

τ is the included angle of the roller cone and is obtained by iteration of

$$\frac{\tau}{2} = \tan^{-1} \left\{ \frac{d}{E} \sin \left(\theta - \frac{\tau}{2} \right) \right\} \quad (16)$$

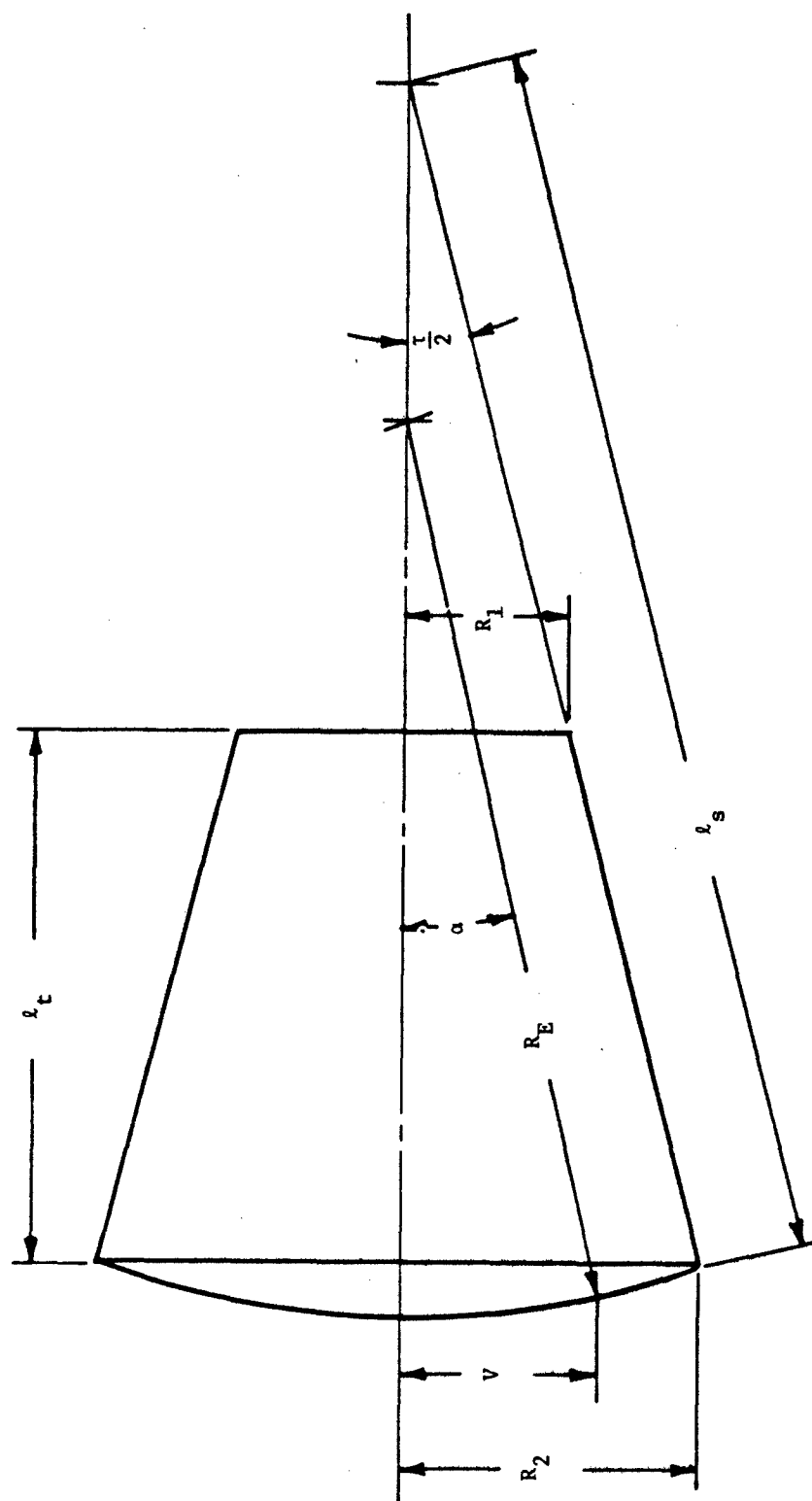


Figure 6. Boundary Dimensions of Typical Tapered Roller

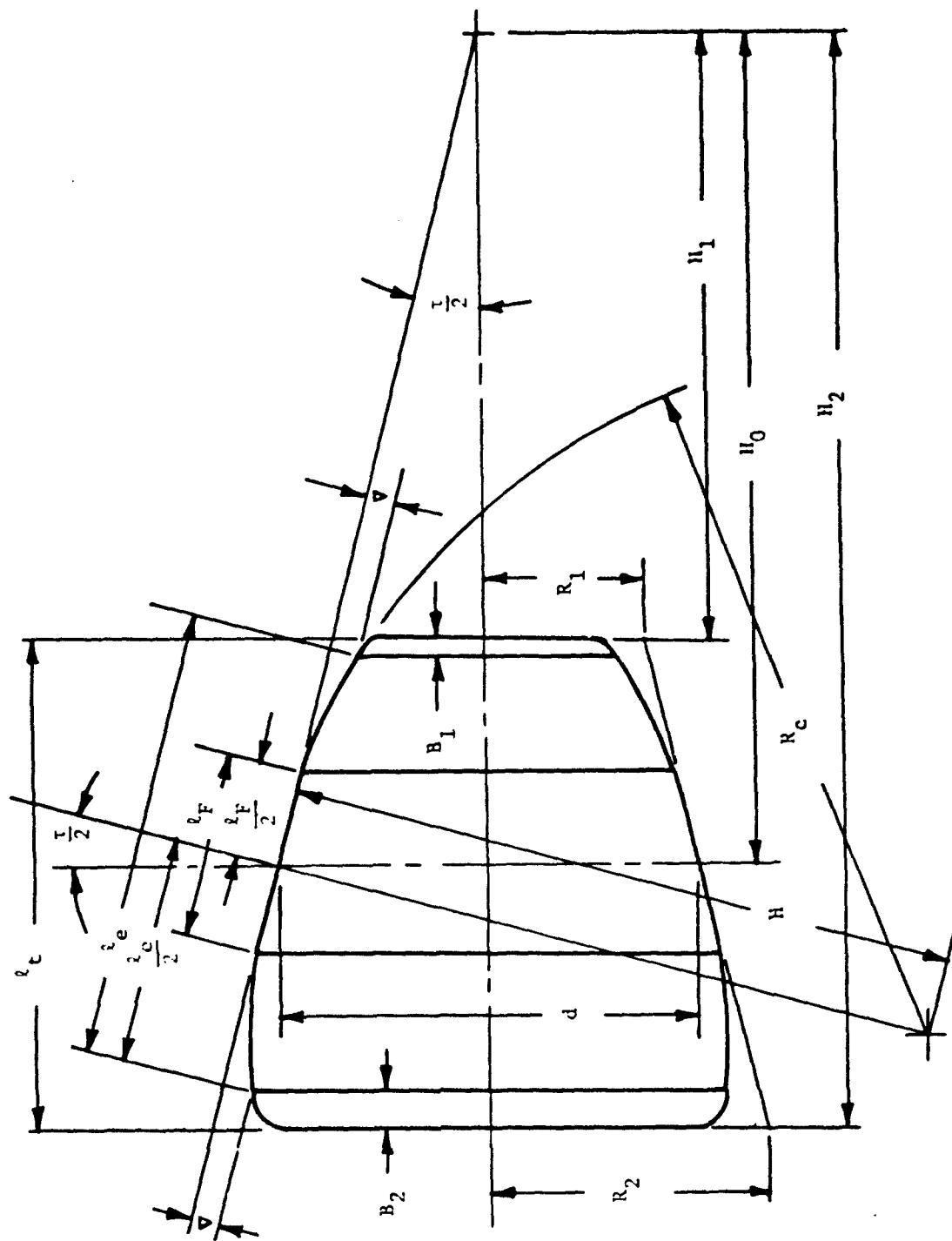


Figure 7. Dimensions of Roller Profile and Crown

From Figure 7

$$H = \sqrt{R_c^2 - \left(\frac{l_F}{2}\right)^2} \quad (17)$$

where R_c is the crown radius and l_F the length of the flat portion of the roller profile. In a fully crowned roller, the flat length is zero.

l_e is the effective length of the roller load-carrying surface. The actual working length for any loading must lie within l_e . B_1 and B_2 are the corner breaks. Their shapes are unimportant as long as they blend smoothly into the crowned surface.

∇ is the drop of the crown and is measured at the extremes of the effective length of the roller.

$$\nabla = H - \sqrt{R_c^2 - \left(\frac{l_e}{2}\right)^2} \quad (18)$$

$$H_0 = \frac{d}{2 \tan\left(\frac{\tau}{2}\right)} \quad (19)$$

$$H_1 = H_0 - \frac{l_e}{2} \cos\left(\frac{\tau}{2}\right) + \nabla \sin\left(\frac{\tau}{2}\right) - B_1 \quad (20)$$

$$H_2 = H_0 + \frac{l_e}{2} \cos\left(\frac{\tau}{2}\right) + \nabla \sin\left(\frac{\tau}{2}\right) + B_2 \quad (21)$$

$$R_1 = H_1 \tan\left(\frac{\tau}{2}\right) \quad (22)$$

$$R_2 = H_2 \tan\left(\frac{\tau}{2}\right) \quad (23)$$

Let the cone corresponding to H_1 have the mass m_1 and a moment of inertia about its center of gravity $I_{1_{cg}}$.

Let the cone corresponding to H_2 have the mass m_2 and a moment of inertia about its center of gravity $I_{2_{cg}}$.

$$m_1 = \frac{\pi R_1^2 H_1 \rho}{3 \times 386.4} \quad (24)$$

$$m_2 = \frac{\pi R_2^2 H_2 \rho}{3 \times 386.4} \quad (25)$$

$$I_{1_{cg}} = \frac{3m_1}{5} \left(\frac{R_1^2}{4} + \frac{H_1^2}{16} \right) \quad (26)$$

$$I_{2_{cg}} = \frac{3m_2}{5} \left(\frac{R_2^2}{4} + \frac{H_2^2}{16} \right) \quad (27)$$

where ρ is the material density in lb/in³.

Then the distance \bar{X} from the big end of the roller at H_2 to the center of gravity of the roller is \bar{X}'

$$\bar{X}' = \frac{\frac{m_2 H_2}{4} - m_1 \left(H_2 - \frac{3H_1}{4} \right)}{m_2 - m_1} \quad (28)$$

The moment of inertia I_{cg} of the tapered roller about its center of gravity at \bar{X} is

$$I_{cg} = I_{2_{cg}} + m_2 \left(\frac{H_2}{4} - \bar{X}' \right)^2 - I_{1_{cg}} - m_1 \left(H_2 - \frac{3H_1}{4} - \bar{X}' \right)^2 \quad (29)$$

Later the distance \bar{X} , being the distance left from H_0 to the center of gravity of the roller, will be required.

$$\bar{X} = H_2 - H_0 - \bar{X}' \quad (30)$$

The slant height, ℓ_s , of the truncated roller cone is shown in Figure 6 and is

$$\ell_s = \frac{R_2}{\sin\left(\frac{\tau}{2}\right)} \quad (31)$$

and the big-end spherical radius, R_E , is a proportion of ℓ_s .

V is the radius from the roller centerline to the point of contact of the roller and inner-race guide flange and the flange reaction. It is directed at an angle, α , where

$$\alpha = \sin^{-1} \left(\frac{V}{R_e} \right) \quad (32)$$

The lever arm of the flange reaction about the midpoint of the working surface of the roller at H_0 is

$$l = \left[\sqrt{R_E^2 - R_2^2} - H_2 + H_0 \right] \sin \alpha \quad (33)$$

Figure 8 is an enlarged view of the race profile showing the crown drop ∇ which is measured at a distance G from the end of the effective length. The contour is the same at both ends of the roll. If the radius R_c is known, the drop at G is

$$\nabla = H - \sqrt{R_c^2 - \left(\frac{l_e}{2} - G \right)^2} \quad (34)$$

If the drop is known and the radius R_c is not, the radius is

$$R_c = \sqrt{\left[\frac{\left(\frac{l_e}{2} - G \right)^2 - \left(\frac{l_F}{2} \right)^2 - \nabla^2}{2\nabla} \right]^2 + \left(\frac{l_e}{2} - G \right)^2} \quad (35)$$

2.4.3 Roller Equilibrium

Figure 9 shows the forces and moments acting on a roller which is in contact with both outer and inner races and with the inner ring guide flange.

In the following discussion, the subscripts 1 and 2 refer to the outer and inner contacts, respectively.

P_1 and P_2 are the contact loads. M_1 and M_2 are contact moments resulting from nonuniform loading along the roller's length. F_c is the centrifugal force and M_G is the gyroscopic moment. The

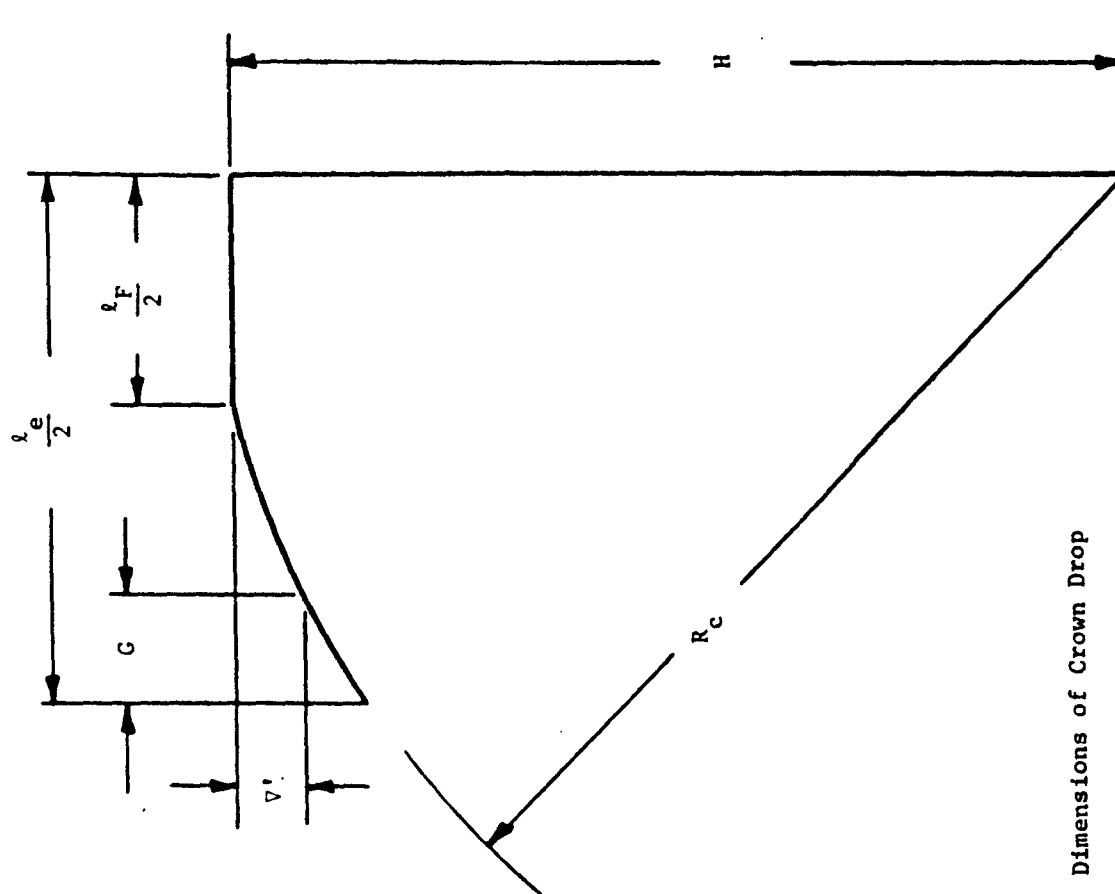


Figure 8. Dimensions of Crown Drop

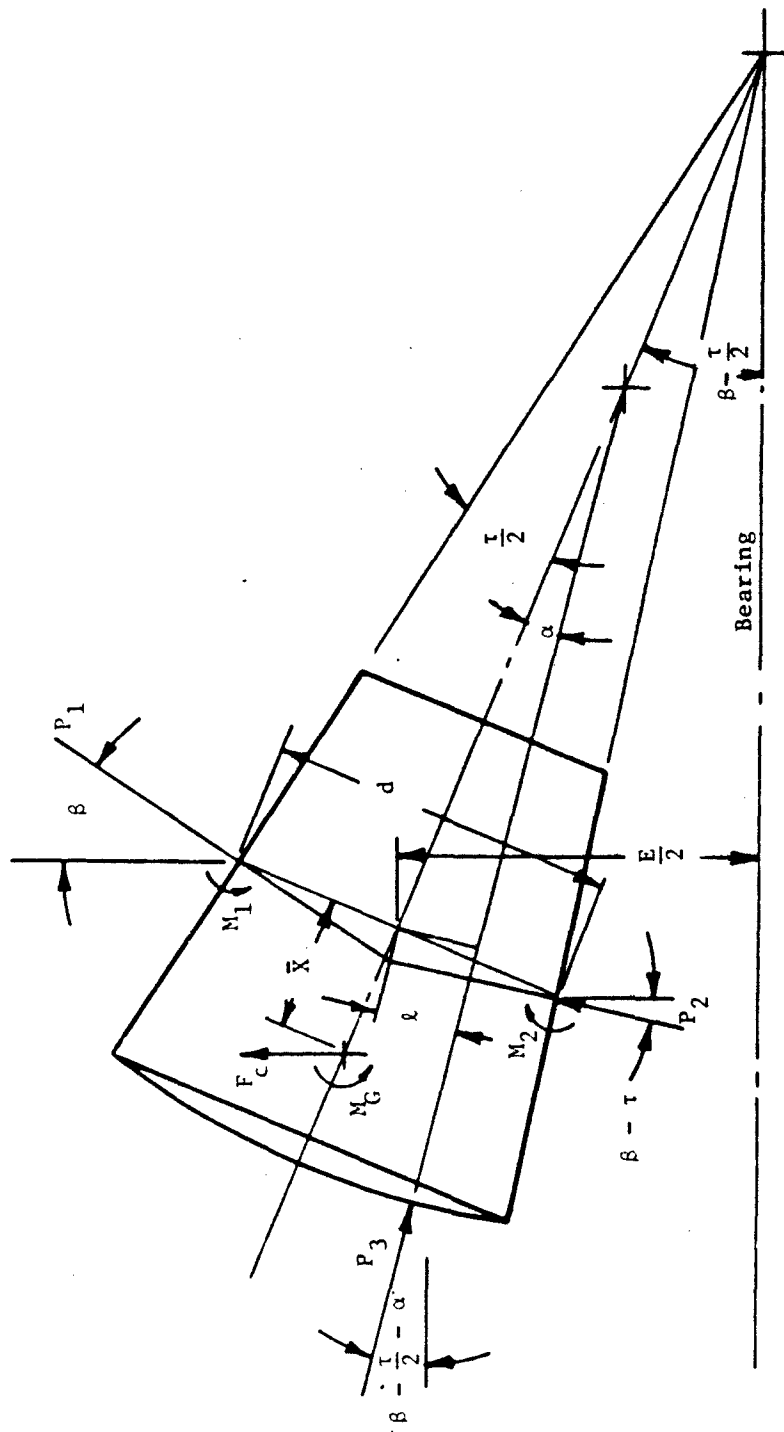


Figure 9. Forces and Moments on Roller

latter acts at the center of gravity of the roller which is located the distance \bar{X} from the central plane of the roller which contains the midpoint of the effective length.

The centrifugal force and the gyroscopic moment are

$$F_c = (m_1 + m_2) \left\{ \frac{E}{2} + \bar{X} \sin\left(\beta - \frac{\tau}{2}\right) \right\} \Omega_E^2 \quad (36)$$

$$M_G = I_{cg} \Omega_E \omega_R \sin\left(\beta - \frac{\tau}{2}\right) \quad (37)$$

where Ω_E is the orbital velocity of the roller and ω_R the angular velocity of the roller about its own center, both in radians/second.

$$\Omega_E = \frac{1}{2} \left[\Omega_1 \left(1 + \frac{d \cos\left(\beta - \frac{\tau}{2}\right)}{E} \right) + \Omega_2 \left(1 - \frac{d \cos\left(\beta - \frac{\tau}{2}\right)}{E} \right) \right] \quad (38)$$

$$\omega_R = \frac{E}{2d} \left[(\Omega_1 - \Omega_2) \left(1 - \frac{d \cos\left(\beta - \frac{\tau}{2}\right)}{E} \right)^2 \right] \quad (39)$$

Ω_1 and Ω_2 are the input angular velocities of outer and inner rings in radians/second. P_3 is the reaction of the inner-ring flange on the roller.

In the present problem, we are concerned with external forces applied to the bearing inner ring along x and/or z (Figure 4) only. There may also be initial linear displacements along any or all of the coordinate axes, x, y, and z; and initial rotations about x and y. These initial displacements do not change when external forces are applied along x and/or z. However, when initial rotations are present about x or z, operating displacements may occur along x and/or y as the case may be. The system, therefore, has the possibility of three degrees of freedom; i.e., working linear displacements along any or all of the axes x, y, and z. If initial displacements exist about x or y, working displacements in these modes are prevented.

The approach of the inner race to the outer race along the line defined by β for a roller at azimuth ϕ is

$$\Delta = (\delta_z + \delta_z'')\sin\beta + \{(\delta_x + \delta_x'')\cos\phi + (\delta_y + \delta_y'')\sin\phi - \frac{P_D}{2}\} \cos\beta + \frac{1}{2}\{E\sin\beta + d\sin(\frac{\tau}{2})\}\{(\theta_x + \theta_x'')\sin\phi + (\theta_y + \theta_y'')\cos\phi\} \quad (40)$$

P_D is the diametral clearance or the total diametral play of the inner ring relative to the outer ring before loading.

The azimuth angle, ϕ , is related to the roller position index, q , through

$$\phi = \frac{2\pi(q-1)}{n} \quad (41)$$

where n is the number of rollers.

The double-primed items in Equation (40) are the initial displacements in the several modes.

Also, as a result of the initial misalignments which may exist about x and/or y , the inner race at the q th roller may be misaligned the amount θ .

$$\theta = (\theta_x + \theta_x'')\sin\phi + (\theta_y + \theta_y'')\cos\phi \quad (42)$$

If Δ_1 is the approach of the roller to the midpoint of the outer race, the approach Δ_2 of the inner ring to the roller at its midpoint is

$$\Delta_2 = \frac{(\Delta - \Delta_1)\cos(\alpha - \frac{\tau}{2})}{\cos(\alpha + \frac{\tau}{2})} \quad (43)$$

If θ_1 is the misalignment of the roller relative to the outer race, the misalignment θ_2 of the inner race relative to the roller is

$$\theta_2 = 0 - \theta_1 \quad (44)$$

Misalignment is positive if it tends to squeeze the big end of the roller more than the little end when the big end is at the left.

Figure 10 illustrates the geometric intersection of a roller and raceway.

The profiles of race and roller bodies are referred to an XY coordinate system. Note that the X axis is positive to the left of the origin.

The equation of the race surface is

$$Y = 0 \quad (45)$$

The equation of the flat portion of the roller or the element of the basic roller cone is

$$Y = \Delta_1 + X \tan \theta_1 \quad (46)$$

The equation of the crowned portion of the roller profile is

$$(X - H \sin \theta_1)^2 + (Y + H \cos \theta_1 - \Delta_1)^2 = R_c^2 \quad (47)$$

The subscript 1 is 1 for an outer contact and 2 for an inner contact.

The intersections of the race and the crowned roller surface occur at X_{A_i} and X_{B_i}

$$X_{A_i} = \sqrt{R_c^2 - (H \cos \theta_1 - \Delta_1)^2} + H \sin \theta_1 \quad (48)$$

$$X_{B_i} = -\sqrt{R_c^2 - (H \cos \theta_1 - \Delta_1)^2} + H \sin \theta_1 \quad (49)$$

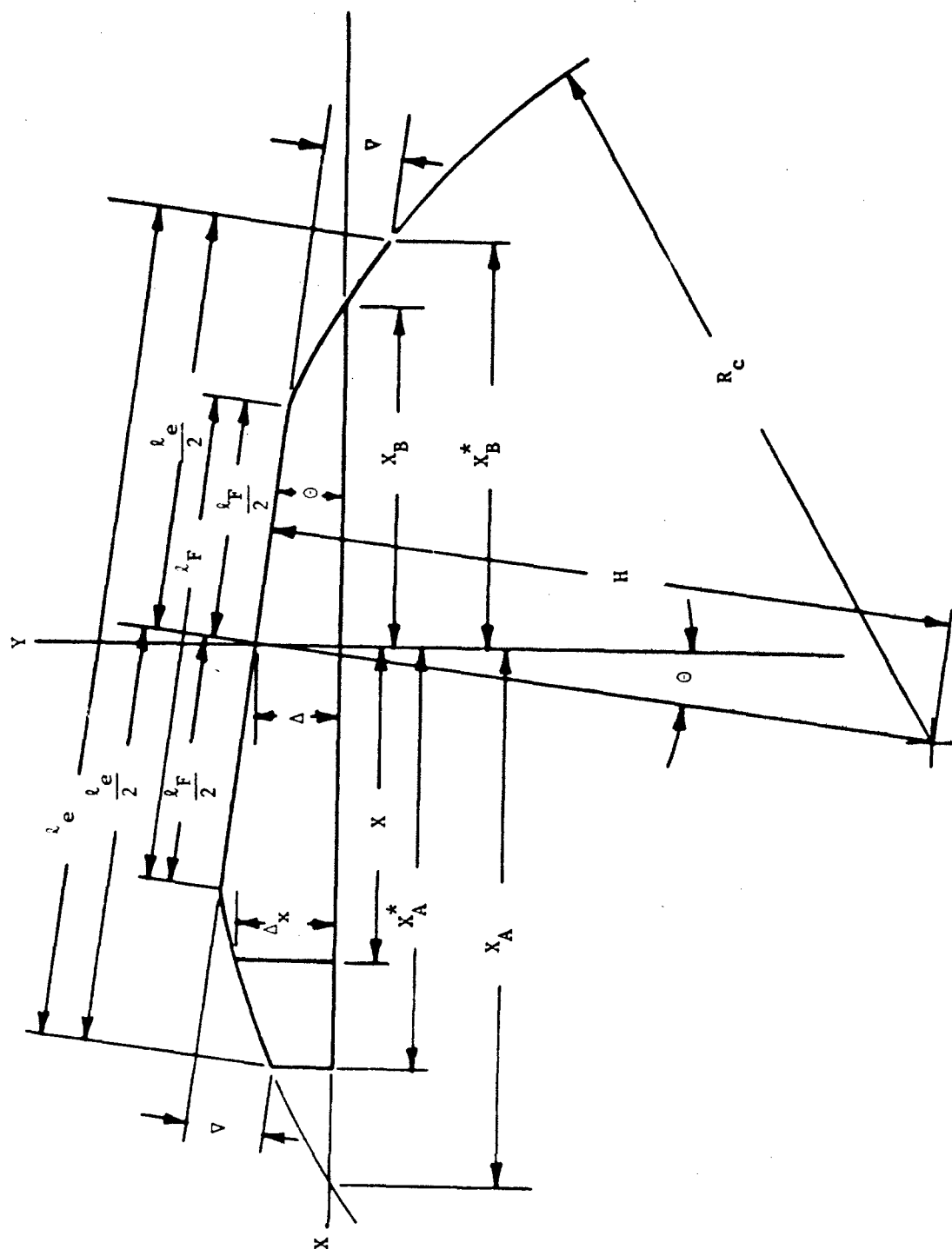


Figure 10. Geometric Intersection of a Roller and Raceway

X_{A_1} and X_{B_1} must be within the projected extremities of the roller crown. That is

$$X_{A_1} \leq X_{A_1}^* \quad (50)$$

$$X_{B_1} \geq X_{B_1}^* \quad (51)$$

where

$$X_{A_1}^* = \frac{l_e}{2} \cos \theta_1 + v \sin \theta_1 \quad (52)$$

$$X_{B_1}^* = -\frac{l_e}{2} \cos \theta_1 + v \sin \theta_1 \quad (53)$$

If the quantity under the radical in Equations (48) and (49) is zero or negative, there is no contact between roller and race.

If $\frac{l_F}{2} \cos \theta_1 \geq X_{A_1}$, there is also no contact.

If $X_{A_1} > X_{A_1}^*$, X_{A_1} is set equal to $X_{A_1}^*$.

If $X_{B_1} < X_{B_1}^*$, X_{B_1} is set equal to $X_{B_1}^*$.

If $\frac{l_F}{2} \cos \theta_1 > X_{B_1} > -\frac{l_F}{2} \cos \theta_1$ and $X_{A_1} > \frac{l_F}{2} \cos \theta_1$,

$$\text{the value of } X_{B_1} \text{ is } X_{B_1} = -\frac{\Delta_1}{\tan \theta_1}. \quad (54)$$

From Figure 9 the conditions for roller force equilibrium are

$$-P_1 \cos \beta + P_2 \cos(\beta - \tau) - P_3 \sin(\beta - \frac{\tau}{2} - \alpha) + F_c = 0 \quad (55)$$

$$-P_1 \sin \beta + P_2 \sin(\beta - \tau) + P_3 \cos(\beta - \frac{\tau}{2} - \alpha) = 0 \quad (56)$$

Equations (55) and (56) are a set of simultaneous nonlinear equations in which the variables are Δ_1 and θ_1 at the outer contact of the

particular roller.

The flange reaction P_3 is obtained by taking moments about the roller midpoint.

$$P_3 = \frac{\{-M_1 + M_2 - M_G + F_c \bar{X} \cos(\beta - \frac{\tau}{2}) - \frac{d}{2} (P_1 - P_2) \sin(\frac{\tau}{2})\}}{l} \quad (57)$$

From Figure 10 the intrusion of the roller into the race is

$$\Delta_x = \Delta_i + X \tan \theta_i \quad |X| \leq \frac{l_F}{2} \cos \theta_i \quad (58)$$

$$\Delta_x = \sqrt{R_c^2 - (X - H \sin \theta_i)^2} - H \cos \theta_i + \Delta_i \quad |X| > \frac{l_F}{2} \cos \theta_i \quad (59)$$

The derivatives of Δ_x with respect to θ_i will be required later and are

$$\frac{d\Delta_x}{d\theta_i} = \frac{X}{\cos^2 \theta_i} \quad |X| \leq \frac{l_F}{2} \cos \theta_i \quad (60)$$

$$\frac{d\Delta_x}{d\theta_i} = \frac{(X - H \sin \theta_i) H \cos \theta_i}{\sqrt{R_c^2 - (X - H \sin \theta_i)^2}} + H \sin \theta_i \quad |X| > \frac{l_F}{2} \cos \theta_i \quad (61)$$

Lundberg (6) gives the approach Δ_x of two cylindrical bodies pressed together with the uniform loading p_x as

$$\Delta_x = \frac{(\eta_R + \eta_E)}{2\pi} p_x \{1.8864 + \ln \left(\frac{X_A - X_B}{2b_x} \right)\} \quad (62)$$

η_R and η_E are elastic constants for race and roller, respectively, having the form

$$\eta_{R,E} = \frac{4(1 - \nu^2)}{E_{R,E}} \quad (63)$$

where ν is Poisson's Ratio and E is the modulus of elasticity.

b_x is the semi-width of the pressure area in the rolling direction.

$$b_x = \left[\frac{(\eta_R + \eta_E)}{2\pi} p_x d_x (1 + C_i \gamma_i) \right]^{1/2} \quad (64)$$

C_i is 1 for $i = 1$, corresponding to an outer contact; and -1 for $i = 2$, corresponding to an inner contact.

$$\gamma_1 = \frac{d_x \cos \beta}{E_x} \quad (65)$$

$$\gamma_2 = \frac{d_x \cos(\beta - \tau)}{E_x} \quad (66)$$

where

$$d_x = \frac{d + 2X \sin(\frac{\tau}{2})}{\cos(\frac{\tau}{2})} \quad (67)$$

$$E_x = E + 2X \sin \beta + d \cos(\beta - \frac{\tau}{2}) - d_x \cos \beta \quad (68)$$

The value of p_x corresponding to Δ_x is required. This cannot be obtained from Equation (62) in closed form. It can be obtained numerically in the following manner.

Let p'_x be an estimate of p_x . A good starting value is

$$p'_x = \frac{5 \times 10^7 \Delta_x^{10/9}}{(X_A - X_B)^{1/9}} \quad (69)$$

An improved value of p_x is

$$p_x = p'_x - \frac{(\Delta'_x - \Delta_x)}{d\Delta'_x/dp'_x} \quad (70)$$

Δ'_x is the approach of race and roller bodies calculated for the current estimate of p'_x using Equation (62).

$d\Delta'_x/dp'_x$ is obtained from Equations (62) and (64) using the current estimate p'_x and is

$$\frac{d\Delta'_x}{dp'_x} = \frac{(\eta_R + \eta_E)}{2\pi} \{1.3864 + \ln \left(\frac{X_A - X_B}{2b_x} \right)\} \quad (71)$$

Iteration of Equation (70) yields p_x to any desired accuracy.

The contact force, P , and the moment, M , are

$$P_i = \int_{X_{B_i}}^{X_{A_i}} p_x dx \quad (72)$$

$$M_i = \int_{X_{B_i}}^{X_{A_i}} x p_x dx \quad (73)$$

Equations (55) and (56) may now be solved for Δ_1 and θ_1 , the displacements at the outer contact. Again, a closed-form solution cannot be obtained and numerical techniques are employed.

If estimates are made of the variables Δ_1 and θ_1 , Equations (55) and (56) may not be satisfied and there will be the residues ϵ_1 and ϵ_2 for Equations (55) and (56), respectively. Differentiating Equations (55) and (56) gives:

$$\frac{d\epsilon_1}{d\Delta_1} = -\cos\beta \frac{dP_1}{d\Delta_1} + \cos(\beta-\tau) \frac{dP_2}{d\Delta_2} \frac{d\Delta_2}{d\Delta_1} - \sin\left(\beta - \frac{\tau}{2} - \alpha\right) \frac{dP_3}{d\Delta_1} \quad (74)$$

$$\frac{d\epsilon_1}{d\theta_1} = -\cos\beta \frac{dP_1}{d\theta_1} + \cos(\beta-\tau) \frac{dP_2}{d\theta_2} \frac{d\theta_2}{d\theta_1} - \sin\left(\beta - \frac{\tau}{2} - \alpha\right) \frac{dP_3}{d\theta_1} \quad (75)$$

$$\frac{d\epsilon_2}{d\Delta_1} = \sin\beta \frac{dP_1}{d\Delta_1} + \sin(\beta-\tau) \frac{dP_2}{d\Delta_2} \frac{d\Delta_2}{d\Delta_1} + \cos\left(\beta - \frac{\tau}{2} - \alpha\right) \frac{dP_3}{d\Delta_1} \quad (76)$$

$$\frac{d\epsilon_2}{d\theta_1} = \sin\beta \frac{dP_1}{d\theta_1} + \sin(\beta-\tau) \frac{dP_2}{d\theta_2} \frac{d\theta_2}{d\theta_1} + \cos(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\theta_1} \quad (77)$$

From Equations (43) and (44)

$$\frac{d\Delta_2}{d\Delta_1} = \frac{-\cos(\alpha - \frac{\tau}{2})}{\cos(\alpha + \frac{\tau}{2})} \quad (78)$$

$$\frac{d\theta_2}{d\theta_1} = -1 \quad (79)$$

And, from Equation (57)

$$\frac{dP_3}{d\Delta_1} = \frac{-\frac{dM_1}{d\Delta_1} + \frac{dM_2}{d\Delta_2} \frac{d\Delta_2}{d\Delta_1} - \frac{d}{2} \left(\frac{dP_1}{d\Delta_1} - \frac{dP_2}{d\Delta_2} \frac{d\Delta_2}{d\Delta_1} \right) \sin(\frac{\tau}{2})}{l} \quad (80)$$

$$\frac{dP_3}{d\theta_1} = \frac{-\frac{dM_1}{d\theta_1} + \frac{dM_2}{d\theta_2} \frac{d\theta_2}{d\theta_1} - \frac{d}{2} \left(\frac{dM_1}{d\theta_1} - \frac{dM_2}{d\theta_2} \frac{d\theta_2}{d\theta_1} \right) \sin(\frac{\tau}{2})}{l} \quad (81)$$

If Δ_1' and θ_1' are current estimates, improved estimates are:

$$\Delta_1 = \Delta_1' - \frac{\begin{vmatrix} \epsilon_1 & \frac{d\epsilon_1}{d\theta_1} \\ \epsilon_2 & \frac{d\epsilon_2}{d\theta_1} \end{vmatrix}}{\begin{vmatrix} \frac{d\epsilon_1}{d\Delta_1} & \frac{d\epsilon_1}{d\theta_1} \\ \frac{d\epsilon_2}{d\Delta_1} & \frac{d\epsilon_2}{d\theta_1} \end{vmatrix}} \quad (82)$$

$$\theta_1 = \theta_1' - \frac{\begin{vmatrix} \frac{d\epsilon_1}{d\Delta_1} & \epsilon_1 \\ \frac{d\epsilon_2}{d\Delta_1} & \epsilon_2 \end{vmatrix}}{\begin{vmatrix} \frac{d\epsilon_1}{d\Delta_1} & \frac{d\epsilon_1}{d\theta_1} \\ \frac{d\epsilon_2}{d\Delta_1} & \frac{d\epsilon_2}{d\theta_1} \end{vmatrix}} \quad (83)$$

The determinants in Equations (82) and (83) are calculated at current estimates.

The derivatives of P_i and M_i with respect to Δ_i and θ_i are

$$\frac{dP_i}{d\Delta_i} = \int_{X_{B_i}}^{X_{A_i}} \frac{dp_x}{d\Delta_x} \frac{d\Delta_x}{d\Delta_i} dX \quad (84)$$

$$\frac{dP_i}{d\theta_i} = \int_{X_{B_i}}^{X_{A_i}} \frac{dp_x}{d\Delta_x} \frac{d\Delta_x}{d\theta_i} dX \quad (85)$$

$$\frac{dM_i}{d\Delta_i} = \int_{X_{B_i}}^{X_{A_i}} X \frac{dp_x}{d\Delta_x} \frac{d\Delta_x}{d\Delta_i} dX \quad (86)$$

$$\frac{dM_i}{d\theta_i} = \int_{X_{B_i}}^{X_{A_i}} X \frac{dp_x}{d\Delta_x} \frac{d\Delta_x}{d\theta_i} dX \quad (87)$$

The value of $dp_x/d\Delta_x$ is obtained from Equation (71) and the value of $d\Delta_x/d\Delta_i$ is unity.

If Equations (43), (44), (55), and (56) are differentiated with respect to Δ , there results four equations which are linear in

$d\Delta_1/d\Delta$, $d\Delta_2/d\Delta$, $d\theta_1/d\Delta$, and $d\theta_2/d\Delta$ and from which all four derivatives can be obtained. Of the four derivatives, only $d\Delta_1/d\Delta$ and $d\theta_1/d\Delta$ are of interest here.

$$\begin{aligned} & \left[-\cos\beta \frac{dP_1}{d\Delta_1} - \sin(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\Delta_1} \right] \frac{d\Delta_1}{d\Delta} + \left[\cos(\beta - \tau) \frac{dP_2}{d\Delta_2} - \right. \\ & \left. \sin(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\Delta_2} \right] \frac{d\Delta_2}{d\Delta} + \left[-\cos\beta \frac{dP_1}{d\theta_1} - \sin(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\theta_1} \right] \\ & \frac{d\theta_1}{d\Delta} + \left[\cos(\beta - \tau) \frac{dP_2}{d\theta_2} - \sin(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\theta_2} \right] \frac{d\theta_2}{d\Delta} = 0 \quad (88) \end{aligned}$$

$$\begin{aligned} & \left[-\sin\beta \frac{dP_1}{d\Delta_1} + \cos(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\Delta_1} \right] \frac{d\Delta_1}{d\Delta} + \left[\sin(\beta - \tau) \frac{dP_2}{d\Delta_2} + \right. \\ & \left. \cos(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\Delta_2} \right] \frac{d\Delta_2}{d\Delta} + \left[-\sin\beta \frac{dP_1}{d\theta_1} + \cos(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\theta_1} \right] \\ & \frac{d\theta_1}{d\Delta} + \left[\sin(\beta - \tau) \frac{dP_2}{d\theta_2} + \cos(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\theta_2} \right] \frac{d\theta_2}{d\Delta} = 0 \quad (89) \end{aligned}$$

$$\frac{d\Delta_1}{d\Delta} + \frac{\cos(\alpha + \frac{\tau}{2})}{\cos(\alpha - \frac{\tau}{2})} \frac{d\Delta_2}{d\Delta} = 1 \quad (90)$$

$$\frac{d\alpha_1}{d\Delta} + \frac{d\alpha_2}{d\Delta} = 0 \quad (91)$$

Equations (88) through (91) are easily solved for $d\Delta_1/d\Delta$ and $d\theta_1/d\Delta$. $d\Delta_1/d\theta$ and $d\theta_1/d\theta$ are obtained in a similar manner.

2.4.4 Bearing Equilibrium

The reactions of the bearing on the shaft at the central plane of the roller are

$$F'_x = \cos\beta \sum_{q=1}^n P_{1q} \cos\phi_q \quad (92)$$

$$F'_y = \cos\beta \sum_{q=1}^n P_{1q} \sin\phi_q \quad (43)$$

$$F'_z = \sin\beta \sum_{q=1}^n P_{1q} \quad (94)$$

$$M'_x = \sum_{q=1}^n \left[\frac{1}{2} \{ E \sin\beta + d \sin(\frac{\tau}{2}) \} P_{1q} + M_{1q} \right] \sin\phi_q \quad (95)$$

$$M'_y = \sum_{q=1}^n \left[\frac{1}{2} \{ E \sin\beta + d \sin(\frac{\tau}{2}) \} P_{1q} + M_{1q} \right] \cos\phi_q \quad (96)$$

Considering the three-degree-of-freedom system, the inner ring is acted upon by the external forces F_x and F_z and may have working displacements along x , y , and z . Equilibrium requires that

$$F'_x + F_x = 0 \quad (97)$$

$$F'_y = 0 \quad (98)$$

$$F'_z + F_z = 0 \quad (99)$$

Here the variables are δ_x , δ_y , and δ_z . Again, a direct solution is not possible and numerical methods must be employed.

For initial estimates δ'_x , δ'_y , δ'_z of the variables Equations (97), (98), and (99) may not be satisfied and there remain the residues ϵ_1 , ϵ_2 , and ϵ_3 . Improved values of the variables are

$$\delta_x = \delta'_x - \frac{\begin{vmatrix} \epsilon_1 & \frac{d\epsilon_1}{d\delta_y} & \frac{d\epsilon_1}{d\delta_z} \\ \epsilon_2 & \frac{d\epsilon_2}{d\delta_y} & \frac{d\epsilon_2}{d\delta_z} \\ \epsilon_3 & \frac{d\epsilon_3}{d\delta_y} & \frac{d\epsilon_3}{d\delta_z} \end{vmatrix}}{D} \quad (100)$$

$$\delta_y = \delta'_y - \frac{\begin{vmatrix} \frac{d\epsilon_1}{d\delta_x} & \epsilon_1 & \frac{d\epsilon_1}{d\delta_z} \\ \frac{d\epsilon_2}{d\delta_x} & \epsilon_2 & \frac{d\epsilon_2}{d\delta_z} \\ \frac{d\epsilon_3}{d\delta_x} & \epsilon_3 & \frac{d\epsilon_3}{d\delta_z} \end{vmatrix}}{D} \quad (101)$$

$$\delta_z = \delta'_z - \frac{\begin{vmatrix} \frac{d\epsilon_1}{d\delta_x} & \frac{d\epsilon_1}{d\delta_y} & \epsilon_1 \\ \frac{d\epsilon_2}{d\delta_x} & \frac{d\epsilon_2}{d\delta_y} & \epsilon_2 \\ \frac{d\epsilon_3}{d\delta_x} & \frac{d\epsilon_3}{d\delta_y} & \epsilon_3 \end{vmatrix}}{D} \quad (102)$$

where D is the determinant of the system.

$$D = \begin{vmatrix} \frac{d\epsilon_1}{d\delta_x} & \frac{d\epsilon_1}{d\delta_y} & \frac{d\epsilon_1}{d\delta_z} \\ \frac{d\epsilon_2}{d\delta_x} & \frac{d\epsilon_2}{d\delta_y} & \frac{d\epsilon_2}{d\delta_z} \\ \frac{d\epsilon_3}{d\delta_x} & \frac{d\epsilon_3}{d\delta_y} & \frac{d\epsilon_3}{d\delta_z} \end{vmatrix} \quad (103)$$

The right members of Equations (100) through (103) are evaluated at current estimates

$$\frac{d\epsilon_1}{d(\delta_x, \delta_y, \delta_z)} = \frac{dF'_x}{d(\delta_x, \delta_y, \delta_z)} \quad (104)$$

$$\frac{d\epsilon_2}{d(\delta_x, \delta_y, \delta_z)} = \frac{dF'_y}{d(\delta_x, \delta_y, \delta_z)} \quad (105)$$

$$\frac{d\epsilon_3}{d(\delta_x, \delta_y, \delta_z)} = \frac{dF'_z}{d(\delta_x, \delta_y, \delta_z)} \quad (106)$$

Although only the above derivatives are required in determining the equilibrium of the system, the complete matrix is required for stiffness calculations.

$$\frac{dF'_y}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} = \cos\beta \sum_{q=1}^n \cos\phi_q \frac{dP_{1q}}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} \quad (107)$$

$$\frac{dF'_y}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} = \cos\beta \sum_{q=1}^n \cos\phi_q \frac{dP_{1q}}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} \quad (108)$$

$$\frac{dF'_z}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} = \sin\beta \sum_{q=1}^n \frac{dP_{1q}}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} \quad (109)$$

$$\begin{aligned} \frac{dM'_x}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} = & \sum_{q=1}^n \left[\frac{1}{2} (E \sin\beta + d \sin(\frac{\tau}{2})) \frac{dP_{1q}}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} + \right. \\ & \left. \frac{dM_{1q}}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} \right] \sin\phi_q \end{aligned} \quad (110)$$

$$\begin{aligned} \frac{dM'_y}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} = & \sum_{q=1}^n \left[\frac{1}{2} (E \sin\beta + d \sin(\frac{\tau}{2})) \frac{dP_{1q}}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} + \right. \\ & \left. \frac{dM_{1q}}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} \right] \cos\phi_q \end{aligned} \quad (111)$$

where

$$\begin{aligned} \frac{dP_{1q}}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} = & \left[\frac{dP_{1q}}{d\Delta_{1q}} \frac{d\Delta_{1q}}{d\Delta_q} + \frac{dP_{1q}}{d\theta_{1q}} \frac{d\theta_{1q}}{d\Delta_q} \right] \frac{d\Delta_q}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} \\ & + \left[\frac{dP_{1q}}{d\Delta_{1q}} \frac{d\Delta_{1q}}{d\theta_q} + \frac{dP_{1q}}{d\theta_{1q}} \frac{d\theta_{1q}}{d\theta_q} \right] \frac{d\theta_q}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} \end{aligned} \quad (112)$$

$$\begin{aligned} \frac{dM_1}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} = & \left[\frac{dM_1}{d\Delta_1} \frac{d\Delta_1}{d\Delta_q} + \frac{dM_1}{d\theta_1} \frac{d\theta_1}{d\Delta_q} \right] \frac{d\Delta_q}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} + \\ & \left[\frac{dM_1}{d\Delta_1} \frac{d\Delta_1}{d\theta_q} + \frac{dM_1}{d\theta_1} \frac{d\theta_1}{d\theta_q} \right] \frac{d\theta_q}{d(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)} \end{aligned} \quad (113)$$

The derivatives of Δ_q and θ_q with respect to the inner-ring displacements are, from Equations (40) and (42)

$$\frac{d\Delta_q}{d\delta_x} = \cos\beta \cos\phi_q \quad (114)$$

$$\frac{d\Delta_q}{d\delta_y} = \cos\beta \sin\phi_q \quad (115)$$

$$\frac{d\Delta_q}{d\delta_z} = \sin\beta \quad (116)$$

$$\frac{d\Delta_q}{d\theta_x} = \frac{1}{2} \{ E \sin\beta + d \sin(\frac{\tau}{2}) \} \sin\phi_q \quad (117)$$

$$\frac{d\Delta_q}{d\theta_y} = \frac{1}{2} \{ E \sin\beta + d \sin(\frac{\tau}{2}) \} \cos\phi_q \quad (118)$$

$$\frac{d\theta_q}{d(\delta_x, \delta_y, \delta_z)} = 0 \quad (119)$$

$$\frac{d\theta_q}{d\theta_x} = \sin\phi_q \quad (120)$$

$$\frac{d\theta_q}{d\theta_y} = \cos\phi_q$$

2.4.5 Effect of Unloaded Roller

In some instances one or more rollers may be out of contact with the inner race while in contact with the outer race and the inner-ring flange. The conditions for equilibrium of such rollers are

$$-P_1 \cos \beta - P_3 \sin(\beta - \frac{\tau}{2} - \alpha) + F_c = 0 \quad (121)$$

$$-P_1 \sin \beta + P_3 \cos(\beta - \frac{\tau}{2} - \alpha) = 0 \quad (122)$$

where

$$P_3 = \frac{-M_1 - M_G - \frac{1}{2} P_1 d \sin(\frac{\tau}{2}) + F_c \bar{X} \cos(\beta - \frac{\tau}{2})}{L} \quad (123)$$

Here the variables are Δ_1 and θ_1 . Initial estimates Δ_1' and θ_1' will generally fail to satisfy Equations (121) and (122), and there will be the residues ϵ_1 and ϵ_2 .

Improved values are

$$\Delta_1 = \Delta_1' - \frac{\begin{vmatrix} \epsilon_1 & \frac{d\epsilon_1}{d\theta_1} \\ \epsilon_2 & \frac{d\epsilon_2}{d\theta_1} \end{vmatrix}}{\begin{vmatrix} \frac{d\epsilon_1}{d\Delta_1} & \frac{d\epsilon_1}{d\theta_1} \\ \frac{d\epsilon_2}{d\Delta_1} & \frac{d\epsilon_2}{d\theta_1} \end{vmatrix}} \quad (124)$$

$$\theta_1 = \theta_1' - \frac{\begin{vmatrix} \frac{d\epsilon_1}{d\Delta_1} & \epsilon_1 \\ \frac{d\epsilon_2}{d\Delta_1} & \epsilon_2 \end{vmatrix}}{\begin{vmatrix} \frac{d\epsilon_1}{d\Delta_1} & \frac{d\epsilon_1}{d\theta_1} \\ \frac{d\epsilon_2}{d\Delta_1} & \frac{d\epsilon_2}{d\theta_1} \end{vmatrix}} \quad (125)$$

The right members of Equations (124) and (125) are evaluated at current estimates. Iteration of Equations (124) and (125) yield Δ_1 and θ_1 to any desired accuracy.

The derivatives required in Equations (124) and (125) are

$$\frac{d\epsilon_1}{d\Delta_1} = -\cos\beta \frac{dP_1}{d\Delta_1} - \sin(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\Delta_1} \quad (126)$$

$$\frac{d\epsilon_1}{d\theta_1} = -\cos\beta \frac{dP_1}{d\theta_1} - \sin(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\theta_1} \quad (127)$$

$$\frac{d\epsilon_2}{d\Delta_1} = -\sin\beta \frac{dP_1}{d\Delta_1} + \cos(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\Delta_1} \quad (127)$$

$$\frac{d\epsilon_2}{d\theta_1} = -\sin\beta \frac{dP_1}{d\theta_1} + \cos(\beta - \frac{\tau}{2} - \alpha) \frac{dP_3}{d\theta_1} \quad (128)$$

where

$$\frac{dP_3}{d\Delta_1} = \frac{-\frac{dM_1}{d\Delta_1} - \frac{1}{2} d\sin(\frac{\tau}{2}) \frac{dP_1}{d\Delta_1}}{\ell} \quad (129)$$

$$\frac{dP_3}{d\theta_1} = \frac{-\frac{dM_1}{d\theta_1} - \frac{1}{2} d\sin(\frac{\tau}{2}) \frac{dP_1}{d\theta_1}}{\ell} \quad (130)$$

Rollers which are out of contact with the inner race must be considered in evaluating the bearing's reactions. They, however, contribute nothing to the stiffness matrix since P_1 and M_1 for these rollers do not change with changes in the inner-ring displacements.

SECTION III

APPLICATION OF COMPUTER PROGRAM

The analysis of Section II has been programmed in Fortran IV for a digital computer and is suitable for use on the CDC 6600. A program listing is presented in the Appendix.

3.1 Sample Test Case

To illustrate a typical case consider the bearing in Figure 11. This is a tapered roller bearing assembly modified for high speed operation. The geometry of this sample bearing is summarized below.

Number of rollers	37
Roller diameter at midpoint	.2913 in.
Pitch diameter	5.0 in.
Contact angle at outer race	14°40'
Effective length of roller	.6001 in.
Roller big-end spherical radius	0.8 in.
Radius from roller centerline to point of big-end spherical surface with inner race flange	0.75 in.
Roller crown radius	100 in.
Roller small-end corner break	.02 in.
Roller big-end corner break	.03 in.
Crown drop gage point	.03 in.

The operating conditions for the sample case are:

Rotational speed	=	20,000 rpm
Load Condition #1		
Thrust Load	=	3,000 lbs.
Load Condition #2		
Thrust Load	=	3,000 lbs.
Radial Load	=	700 lbs.

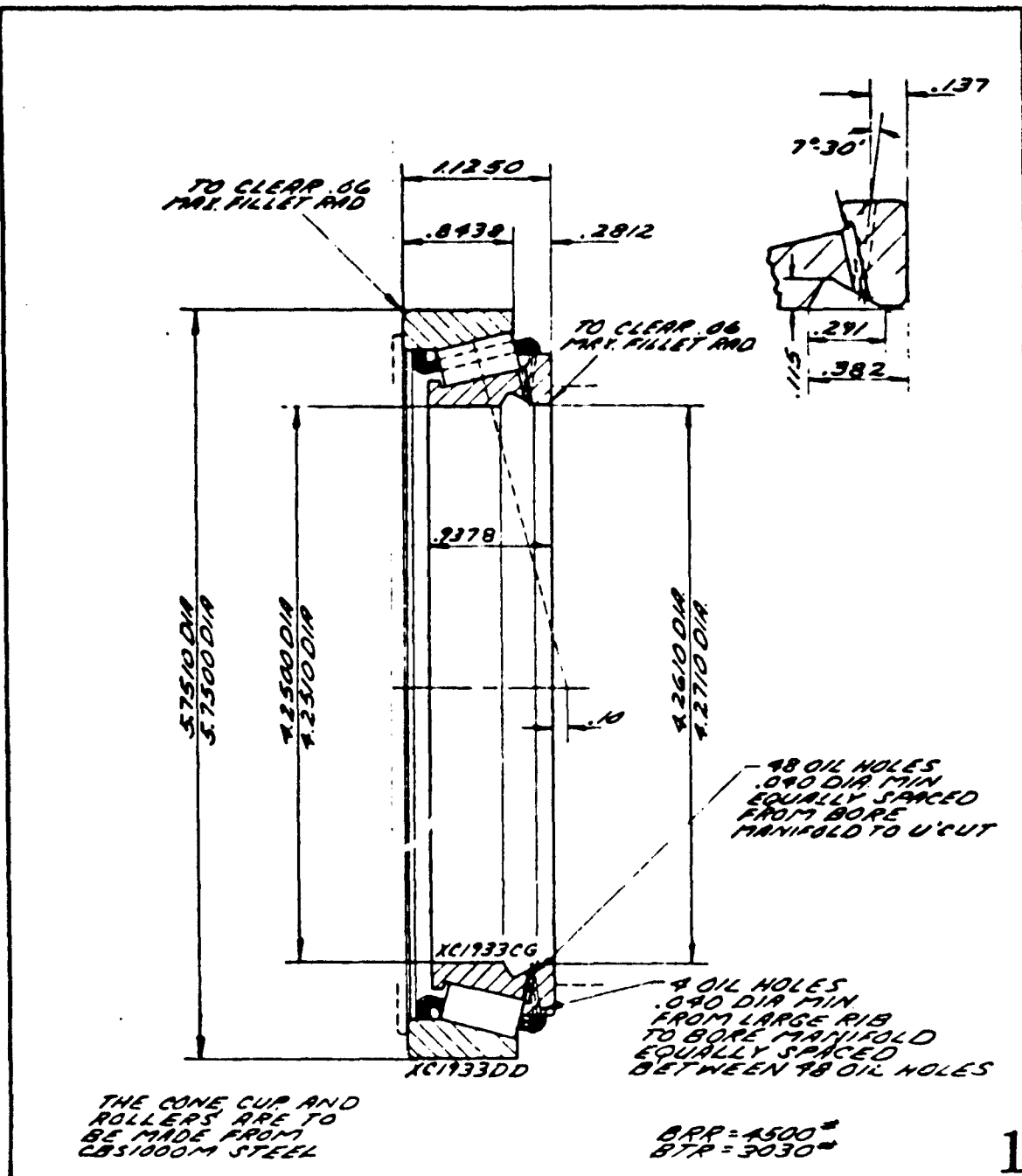


Figure 11. Sample Tapered Roller Bearing Assembly

3.2 Input Format

Figure 12 presents the input data format and Figure 13 shows the actual input data for Load Conditions #1 and #2 of the sample case.

3.3 Output Format

Figure 14 presents the output data for Load Condition #1. The input data are summarized in Figure 14, followed by the output data including the internal load distribution as well as various other stress and displacement parameters. The stiffness matrix is given on the last page of Figure 14.

The output data for Load Condition #2 are presented in Figure 15.

Number of Rolls (60 maximum)	Roll Diameter at Midpoint - Inches	Pitch Diameter - Inches	Contact Angle at Outer Race - Degrees (Must be Positive)	Total Length of Roll - Inches	Effective Length of Roll - Inches	Length of Flat Portion of Roll Working Surface - Inches	Roll Big End Surface Spherical Radius - Inches
TITLE							
TITLE							
Radius from Roll Centerline to Point of Big End Spherical Inner Race Flange	Roll Crown Radius - Inches	Roll Crown Drop - Inches	Roll Small End Corner P Rear Inches	Roll Big End Corner Break Inches	Crown Drop Gage Point - Inches	Flange Clearance Inches	Roll Material Density lb/in ³
Modulus of Elasticity Outer Ring - lb/in ²	Modulus of Elasticity Inner Ring - lb/in ²	Modulus of Elasticity Rolls - lb/in ²	Poisson's Ratio Outer Ring (if Blank Assumed 0.25)	Poisson's Ratio Inner Ring	Poisson's Ratio Rolls		
RPM - Outer Ring	RPM - Inner Ring	Force Along x - Lb	Force Along y - Lb Must be Negative	Initial Dis- placement Along x - Inches	Initial Dis- placement Along y - Inches	Initial Dis- placement Along z - Inches	Initial Dis- placement About x - Radians
Initial Dis- placement About y - Radians							

E 10.0 Format

- (A) If total length is given omit effective length
 If effective length is given omit total length
 (B) Enter 1 to start printout at top of new page
 (C) If crown radius is given omit crown drop. If drop is given omit radius.
 To run additional load cases with same bearing, repeat cards 6 and 7 directly after last card 7.
 To run new system place 2 blanks after last card 7 and repeat cards 1, etc.

Figure 12. Input Data Format

IBM

GLS-727-6 U/M 090**
Printed in U.S.A.

FORTRAN Coding Form

TAPERED ROLLER BEARING STIFFNESS

J. BARTLEY

FEB 1979

Page 1 of 1

Line 1

Line	Statement	Column	Row
37	2913159 5	14.66667	.6001
1	SAMPLE PROBLEM - TAPERED ROLLER BEARING		
	FULLY-CROWNED ROLLS	.02	.03
75	100		
	BLANK CARD	-3000	
	20000	-3000	
	20000		
	700		
	BLANK CARD		
	BLANK CARD		
	BLANK CARD		

Figure 13. Sample Problem Data Input

SAMPLE PROBLEM - TAPERED ROLLER PLATING
FULLY-CROWNED ROLLS

DESIGN DATA FOR BEARING NO. 1

NO. OF ROLLS	ROLL DIAMETER IN	PITCH DIAMETER IN	CONTACT ANGLE DEG	TOTAL LENGTH IN	EFFECTIVE LENGTH IN	FLAT LENGTH IN	VALUE OF ϵ_1 IN	VALUE OF ϵ_2 IN	CROWN RADIUS IN	CROWN CROP IN
3.7000-01	2.9132-01	5.0000-00	1.4567-01	6.5004-01	6.3010-01	0.0000	2.0000-02	3.0000-02	1.0000-02	3.0000-00
VALUE OF ϵ_2 IN	SPHERICAL END IN	INCLUDED ANGLE DEG	VALUE OF ANGLE DEG	LOCATION OF CENTER IN	ROLL LENGTH IN	MOD. OF IN. ROLL C-2	ROLL LENGTH IN	MODULUS OF ELASTICITY LB/IN ²	OUTER RADIUS IN	OUTER CROWN IN
1.1270-01	8.6699-00	1.8000-00	7.5500-01	1.1751-02	1.2277-02	1.0000-00	2.0000-01	2.0000-01	2.0000-01	2.0000-01
OUTER RADIUS IN	POISSON'S RATIO	ROLLS	DIAMETRAL CLEARANCE IN							
2.5000-01	2.5000-01	2.5000-01	0.0000							

Figure 14. Output Data for Load Condition #1

```

INPUT DATA FOR LOAD NO. 1 BEARING NO. 1
RPM OF 1.0000
LOADS APPLIED TO INNER
ALONG X 0.0000
ALONG Y 0.0000
ALONG Z -1.0000
CENTRIFUGAL FORCE 0.0000
REFLECTION PERMITTING 1.0000
ALONG X 0.0000
ALONG Y 0.0000
ALONG Z 0.0000
OUTPUT DATA FOR LOAD NO. 1 BEARING NO. 1
REACTIONS OF BEARING ON SHAFT
ALONG X -6.5340E-04
ALONG Y -4.7684E-04
ALONG Z 3.0001E-03
CONTACT LOAD
CONTACT X 0.0000
CONTACT Y 0.0000
CONTACT Z 0.0000
TOTAL DISPLACEMENTS OF INNER WITH RESPECT TO OUTER
ALONG X 0.0000
ALONG Y 0.0000
ALONG Z 1.9992E-03
QUAVER PATH EXTREMITY
X(1) 0.0000
X(2) 0.0000
X(3) 0.0000
Y(1) 0.0000
Y(2) 0.0000
Y(3) 0.0000
Z(1) 0.0000
Z(2) 0.0000
Z(3) 0.0000
INNER PATH EXTREMITY
X(1) 0.0000
X(2) 0.0000
X(3) 0.0000
Y(1) 0.0000
Y(2) 0.0000
Y(3) 0.0000
Z(1) 0.0000
Z(2) 0.0000
Z(3) 0.0000
FLANGE LOAD
X(1) 0.0000
X(2) 0.0000
X(3) 0.0000
Y(1) 0.0000
Y(2) 0.0000
Y(3) 0.0000
Z(1) 0.0000
Z(2) 0.0000
Z(3) 0.0000

```

Figure 14. (Continued)

NAME	DATE	CONTACT	LEVEL	STATUS	LOCATION	REMARKS
1	1960	1	1	1	1	1
2	1960	1	1	1	1	1
3	1960	1	1	1	1	1
4	1960	1	1	1	1	1
5	1960	1	1	1	1	1
6	1960	1	1	1	1	1
7	1960	1	1	1	1	1
8	1960	1	1	1	1	1
9	1960	1	1	1	1	1
10	1960	1	1	1	1	1
11	1960	1	1	1	1	1
12	1960	1	1	1	1	1
13	1960	1	1	1	1	1
14	1960	1	1	1	1	1
15	1960	1	1	1	1	1
16	1960	1	1	1	1	1
17	1960	1	1	1	1	1
18	1960	1	1	1	1	1
19	1960	1	1	1	1	1
20	1960	1	1	1	1	1
21	1960	1	1	1	1	1
22	1960	1	1	1	1	1
23	1960	1	1	1	1	1
24	1960	1	1	1	1	1
25	1960	1	1	1	1	1
26	1960	1	1	1	1	1
27	1960	1	1	1	1	1
28	1960	1	1	1	1	1
29	1960	1	1	1	1	1
30	1960	1	1	1	1	1
31	1960	1	1	1	1	1
32	1960	1	1	1	1	1
33	1960	1	1	1	1	1
34	1960	1	1	1	1	1
35	1960	1	1	1	1	1
36	1960	1	1	1	1	1
37	1960	1	1	1	1	1
38	1960	1	1	1	1	1
39	1960	1	1	1	1	1
40	1960	1	1	1	1	1
41	1960	1	1	1	1	1
42	1960	1	1	1	1	1
43	1960	1	1	1	1	1
44	1960	1	1	1	1	1
45	1960	1	1	1	1	1
46	1960	1	1	1	1	1
47	1960	1	1	1	1	1
48	1960	1	1	1	1	1
49	1960	1	1	1	1	1
50	1960	1	1	1	1	1
51	1960	1	1	1	1	1
52	1960	1	1	1	1	1
53	1960	1	1	1	1	1
54	1960	1	1	1	1	1
55	1960	1	1	1	1	1
56	1960	1	1	1	1	1
57	1960	1	1	1	1	1
58	1960	1	1	1	1	1
59	1960	1	1	1	1	1
60	1960	1	1	1	1	1
61	1960	1	1	1	1	1
62	1960	1	1	1	1	1
63	1960	1	1	1	1	1
64	1960	1	1	1	1	1
65	1960	1	1	1	1	1
66	1960	1	1	1	1	1
67	1960	1	1	1	1	1
68	1960	1	1	1	1	1
69	1960	1	1	1	1	1
70	1960	1	1	1	1	1
71	1960	1	1	1	1	1
72	1960	1	1	1	1	1
73	1960	1	1	1	1	1
74	1960	1	1	1	1	1
75	1960	1	1	1	1	1
76	1960	1	1	1	1	1
77	1960	1	1	1	1	1
78	1960	1	1	1	1	1
79	1960	1	1	1	1	1
80	1960	1	1	1	1	1
81	1960	1	1	1	1	1
82	1960	1	1	1	1	1
83	1960	1	1	1	1	1
84	1960	1	1	1	1	1
85	1960	1	1	1	1	1
86	1960	1	1	1	1	1
87	1960	1	1	1	1	1
88	1960	1	1	1	1	1
89	1960	1	1	1	1	1
90	1960	1	1	1	1	1
91	1960	1	1	1	1	1
92	1960	1	1	1	1	1
93	1960	1	1	1	1	1
94	1960	1	1	1	1	1
95	1960	1	1	1	1	1
96	1960	1	1	1	1	1
97	1960	1	1	1	1	1
98	1960	1	1	1	1	1
99	1960	1	1	1	1	1
100	1960	1	1	1	1	1

Figure 14. (Continued)

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY #1-1000 TO 1000

PARTIAL DERIVATIVES OF REACTIONS WITH RESPECT TO DISPLACEMENTS			
DEF/LX	DEF/UY	DEF/LZ	DEF/DALX
LB/LN	LB/LN	LB/LN	LB/RAD
1.5514-07	1.1133-01	-4.5117-01	7.9102-02
DEF/LX	DEF/UY	DEF/LZ	DEF/DALY
LB/LN	LB/LN	LB/LN	LB/RAD
1.3477-01	1.5514-07	3.5645-02	1.0170-07
DEF/LX	DEF/UY	DEF/LZ	DEF/DALZ
LB/LN	LB/LN	LB/LN	LB/RAD
-1.6074-01	6.7383-02	2.1253-06	1.0010-02
DEF/LX	DEF/UY	DEF/LZ	DEF/DALX
LB/LN	LB/LN	LB/LN	LB/RAD
6.9330-02	1.0314-07	4.0771-02	6.8923-06
DEF/LX	DEF/UY	DEF/LZ	DEF/DALY
LB/LN	LB/LN	LB/LN	LB/RAD
1.0143-07	7.6155-02	-2.5516-01	6.2015-02

Figure 14. (Continued)

Figure 15. (Continued)

APPENDIX

COMPUTER PROGRAM
FOR
CALCULATING STIFFNESS MATRIX
OF
TAPERED ROLLER BEARING

1	CARD	COL.	VIEW	NUMBER OF ROLLS - 60 MAXIMUM
1	1-10			ROLL DIAMETER - IN. MEASURED AT MIDDLEPOINT OF EFFECTIVE
1	11-20			LENGTH. SEE COLS. 51-60 OF THIS CARD.
1	21-30			PITCH DIAMETER - IN
1	31-40			CONTACT ANGLE AT OUTER RACE - DEG. MUST BE POSITIVE
1	41-50			TOTAL LENGTH OF ROLL - IN. THE AXIAL DISTANCE BETWEEN THE
1				INTERSECTION OF THE ROLL-END FACES AND THE ROLL-CONE
1				ELEMENT MEASURED BETWEEN SHARP CORNERS
1	51-60			EFFECTIVE LENGTH OF ROLL - IN. THE MAXIMUM WORKING LENGTH
1				OF THE ROLL MEASURED ALONG THE ROLL CONE ELEMENT BETWEEN
1				CORNER BREAKS
1				
1				NOTE IF TOTAL LENGTH IS GIVEN OMIT EFFECTIVE LENGTH
1				IF EFFECTIVE LENGTH IS GIVEN OMIT TOTAL LENGTH
1				
1	61-70			LENGTH OF FLAT PORTION OF ROLL WORKING SURFACE MEASURED
1				ALONG THE CONE ELEMENT - IN. FOR A FULLY-CROWNED ROLL THE
1				FLAT LENGTH IS ZERO
1	71-80			ROLL BIG-END SURFACE SPHERICAL RADIUS - IN. IF NEGATIVE
1				ITS ABSOLUTE VALUE MULTIPLIES THE ROLL CONE SLANT HEIGHT
1				MEASURED FROM THE APEX TO THE SHARP INTERSECTION WITH THE
1				BIG-END SURFACE TO GIVE THE SPHERICAL RADIUS
2	1			PUNCH 1. TO START THE PRINTOUT AT THE TOP OF A NEW PAGE
2	2-80			TITLE CARD. PUNCH ANYTHING
3	1			LEAVE BLANK
3	2-80			TITLE CARD. PUNCH ANYTHING
4	1-10			RADIUS FROM ROLL CENTERLINE TO POINT OF CONTACT OF BIG-
4				END SPHERICAL SURFACE WITH THE INNER-RACE FLANGE - IN.
4				IF NEGATIVE ITS ABSOLUTE VALUE MULTIPLIES THE BIG-END
4				RADIUS FROM THE ROLL CENTERLINE TO THE SHARP INTERSECTION
4				OF THE BIG-END SURFACE AND THE PROJECTED ROLL CONE
4				ELEMENT TO GIVE THE DESIRED RADIUS
4	11-20			ROLL CROWN RADIUS - IN.
4	21-30			ROLL CROWN DROP - IN. MEASURED FROM GAGE POINT. SEE COLS.
4				51-60 OF THIS CARD
4				
4				NOTE IF CROWN RADIUS IS GIVEN OMIT CROWN DROP. IF DROP IS

GIVEN OMIT CROWN RADIUS

31-40 ROLL SMALL-END CORNER BREAK - IN. VIEW ROLL WITH AXIS HORIZONTAL AND WITH BIG END AT LEFT. SMALL-END CORNER BREAK IS THE AXIAL DISTANCE BETWEEN THE SHARP INTERSECTION OF THE SMALL END-SURFACE WITH THE ROLL-CONE ELEMENT AND THE RIGHTMOST BOUND OF THE EFFECTIVE LENGTH ROLL PIG-END CORNER BREAK - IN. VIEW ROLL WITH AXIS HORIZONTAL AND BIG END AT LEFT. PIG-END CORNER BREAK IS THE AXIAL DISTANCE BETWEEN THE SHARP INTERSECTION OF THE RIG-END SPHERICAL SURFACE WITH THE ROLL-CONE ELEMENT AND THE LEFTMOST BOUND OF THE EFFECTIVE LENGTH CROWN DROP GAGE POINT - IN. REFERENCE POINT FOR THE MEASUREMENT OF CROWN DROP - IN. IT IS THE DISTANCE MEASURED ALONG THE PROJECTED ROLL-CONE ELEMENT FROM THE OUTER EDGE OF THE EFFECTIVE LENGTH TO THE POINT OF CROWN DROP MEASUREMENT. IT IS THE SAME FOR BOTH ENDS OF THE ROLL

61-70 DIAMETRAL CLEARANCE - IN. THE TOTAL DIAMETRAL LOOSENESS OR SHAKE IN THE MOUNTED BEARING BEFORE LOADING. A NEGATIVE VALUE INDICATES TIGHTNESS.

71-80 ROLL MATERIAL DENSITY - LB/IN**3
IF BLANK PROGRAM ASSUMES .283

5 1-10 MODULUS OF ELASTICITY FOR OUTER RING - LB/IN**2
11-20 SAME FOR INNER RING
21-30 SAME FOR ROLLS
31-40 POISSON'S RATIO FOR OUTER RING
IF BLANK PROGRAM ASSUMES .25
41-50 SAME FOR INNER RING
51-60 SAME FOR ROLLS

6 1-10 RPM OF OUTER RING
11-20 RPM OF INNER RING
21-30 FORCE ALONG X - LB. OBSERVE SIGN
31-40 FORCE ALONG Y - LB. MUST BE NEGATIVE
41-50 INITIAL DISPLACEMENT ALONG X - IN
51-60 INITIAL DISPLACEMENT ALONG Y - IN
61-70 INITIAL DISPLACEMENT ALONG Z - IN

71-AU INITIAL DISPLACEMENT ABOUT X - RADIANS
 1-10 INITIAL DISPLACEMENT ABOUT Y - RADIANS
 11-20 A 1. HERE PERMITS OPERATING DISPLACEMENTS ALONG X
 21-30 A 1. HERE PERMITS OPERATING DISPLACEMENTS ALONG Y
 31-40 A 1. HERE PERMITS OPERATING DISPLACEMENTS ALONG Z

TO RUN ADDITIONAL LOAD CASES WITH THE SAME BEARING REPEAT
 CARDS 6 AND 7 AS A UNIT DIRECTLY AFTER LAST CARD 7

TO RUN NEW SYSTEM PLACE TWO BLANKS AFTER LAST CARD 7 AND
 REPEAT CARDS 1 ET SEQ.

TO STOP PLACE THREE BLANKS AFTER LAST CARD 7

COMMON ALPHA,AA(5,5),A(5,5),ABIG, BETA,P1,R2,RTA,BMTAU,BMT02,
 1RT2AL,BGTWP,BR(2,60), C(2),CROWN,CBTA,CTAU02,CBMT,U,CBMT02,
 2CRT2AL,CP4,CORR(5),CTH,COR(5), D,DRAD,DELU(5),DFL11(5),DFL(3)
 3,OTV(5,5),DELTA,DEL(2),DPDEL(2),DMDEL(2),DPH(2),DMTH(2),DELX,
 4DOTH,DX,DLX(2,60),DELSAV(2,60),DPID(5),DPID1(5),DELL,DELLX,
 5F,EL(2),EPR(5),EFL,EX,ER(5), FLI,FLTA2,FRFE(3),FC, GAGE,GAM,
 6GMM,GY,GXX, H1,H2,HERTZ(2,60),HR7,H7, IBR,ISTOP,ILOAD,IQUIT,
 7ICT,JPASS, KKK, N,NLOAD, OME,OMD, PR,PR(2),PR,PK(2),
 AP(2,60),P3,PK3(60),PIOUT,P3OUT, RS,RHO,RW,R1,R2,RP"(2)
 COMMON SRTA,STAU02,SLANT,SRMTAU,SBT2AL,SAV1,SAV2,SAV3,SP4,STH,
 1 THD1(5),TAU02,TAU,TTAU02,TAT2AL,TOL(5),THETA,TH(2),TANTH,
 2THSAV(2,60),THET,TMPBIG, V,VV, WEIGHT, XN,XLT,XLE,XFAU,
 3XLT02,XLE02,XNAR,XMASS1,XMASS2,XMCG1,XINCG2,XINCG,YBAR1,
 4XPAR2,XRAR,XLEVER,XALPHA,XF(2),XFI(5),XV(2),XTH,XDEL,X1,X2,
 5XMM(2,60),XX(2,2,60),XMSAV(2,60),XMINUT,XO(2),XHBIG, YN(2),
 6Y"R

DOUBLE PRECISION BGTWP,CTH,DELTA,DEL,DPDEL,DMDEL,DPH,DMTH,GWX,HD
 1,PX,P3,STH,THETA,TH,TANTH,VV,XM,XTH,XDEL
 IRR=0
 C(1)=1.
 C(2)=-1.

```

10 READ(5,20)XN,D,E,BETA,XLT,XLE,FLT,RS
20 FORMAT(8E10.0)
   IF(XN.EQ.0.)STOP
   PFAD(5,30)
30 FORMAT(R04)
   1 /R04
   2 )
   WRITE(6,30)
   READ(5,20)V,CROWN,DROP,R1,B2,GAGE,PD,RHO,YM(1),YV(2),YMR,PR(1),PR(
12),PRR
   ICT=0
   ISTOP=0
   IPR=IPR+1
   ILOAD=0
   BTA=BETA/57.29578
   SRTA=SIN(BTA)
   CRTA=COS(BTA)
   DELD1(1)=SBTA
   THD1(1)=0.
   THD1(2)=0.
   THD1(3)=0.
   N=XN
   YMR=29.E6
   PPR=.25
   DO 40 K=1,2
   YV(K)=29.E6
   PR(K)=.25
40 EL(K)=.636619R*((1.-PPR**2)/YMR+(1.-PR(K)**2)/YV(K))
   IF(RHO.EQ.0.)RHO=.283
   GAM=D/E
   TAU02=0.
   DO 50 ITT=1,30
   TEMP=TAU02
   TAU02=ATAN(GAM*SIN(BTA-TAU02))
   IF(ABS(TAU02-TEMP)-5.E-7)70,50,50
50 CONTINUE
   WRITE(6,60)
60 FORMAT(18H0 MAIN PROGRAM 60)
   ISTOP=1

```

```

70      GO TO 140
        TAU=2.*TAU02
        XTAU=57.29379*TAU
        CTAU02=SI*(TAU02)
        CTAU02=CO5(TAU02)
        TTAU02=CTAU02/CTAU02
        IF (XLT) 90,80,80
        XLT=XLE*CTAU02+R1+32
        GO TO 140
90      XLE=(XLT-R2-R1)/CTAU02
100     XLT02=.5*XLT
        XLE02=.5*XLE
        FLT02=.5*FLT
        CALL PVALC(CROWN,DROP,FLT02,GAGF,H3,XLE02)
        P"4=HN
        XJAB=RMH-SQRT(CROWN**2-XLE02**2)
        H0=.5*J/TTAU02
        H1=.5*J/TTAU02-XLE02*CTAU02+XNAR*STAU02-R1
        H2=.5*J/TTAU02+XLE02*CTAU02+XNAR*STAU02+R2
        R1=H1*TTAU02
        R2=H2*TTAU02
        X"ASS1=2.710139E-3*R1**2+H1*R40
        X"ASS2=2.710139E-3*R2**2+H2*R40
        X"ASS=X"ASS2-X"MASS1
        XTNCG1=.6*X"MASS1*(H1**2/4.+H1**2/16.)
        XTNCG2=.6*X"MASS2*(R2**2/4.+H2**2/16.)
        XPAR1=H1/4.
        XPAR2=H2/4.
        XPAR=(X"MASS2*XPAR2-X"MASS1*(XLT+YPAR1))/Y"MASS
        XTNCG=XTNCG2+Y"MASS2*(YPAR2-XPAR)**2-XTNCG1-X"MASS1*(XLT+XPAR1-XPAR)
1**2
        XPAR=H2-XPAR-H0
        SLANT=R2/STAU02
        IF (V.LT.0.) V=ABS(V)*R2
        IF (RS.LT.0.) RS=ABS(RS)*SLANT
        ALPHA=ASIN(V/RS)
        XALPHA=ALPHA*57.29578
        XLEVER=(H0-H2+SQRT(RS**2-R2**2))*V/RS
        RVTAU=BTA-1AU

```

```

SRMTAU=SIN(BMTAU)
CRMTAU=COS(BMTAU)
RMT02=BTA-TAU02
SRMT02=SIN(RMT02)
CRMT02=COS(RMT02)
GAMM=CBMT02+GAM
RT2AL=BMT02-ALPHA
SR2AL=SIN(RT2AL)
CRT2AL=COS(RT2AL)
TRT2AL=SBT2AL/CRT2AL
WEIGHT=XMASS*186.4
TOL(1)=5.E-7
TOL(2)=5.E-7
TOL(3)=1.E-6
TOL(4)=1.E-6
TOL(5)=1.E-6
WRITE(6,110)I=1
110 FORMAT(30H0 DESIGN DATA FOR BEARING NO.,I3)
WRITE(6,120)
120 FORMAT(129H0 NO. OF ROLL VALUE PITCH CONTACT TOT
1AL EFFECTIVE FLAT DIAMETER DIAMETER ANGLE CROWN L
2 CROWN/129H LENGTH OF B1 OF B2 RADIUS
3 LENGTH OF B1 OF B2 RADIUS
4 DROP/18X,2HIN,10X,2HIN,9X,7HDEG ,7(12H IN ))
WRITE(6,130)XN,D,E,BETA,XLT,XLE,FLT,R1,R2,CROWN,PROP
130 FORMAT(1P11E12.4)
WRITE(6,140)
140 FORMAT(125H0 VALUE SPHERICAL INCLUDED VALUE OF LOCATI
1ON OF ROLL MOM. OF IN. ROLL MODULUS OF ELASTI
2CITY/120H OF V END RADIUS ROLL ANGLE ALPHA CENTRO
3IN WEIGHT ABOUT C.G. DENSITY OUTER INNER I
4 ROLLS/131H IN IN DEG DEG L2/IN**2 L2/IN**2
5N LR LB*IN*SEC**2 LB/IN**2 LB/IN**2 L2/IN**2
6 LB/IN**2)
WRITE(6,130)V,RS,XTAU,XALPHA,XBAR,WEIGHT,XINCG,PHO,YM(1),YM(2),YMR
WRITE(6,150)
150 FORMAT(47H0 POISSON'S RATIO DIAMETRAL/47H 0
1UTER INNER ROLLS CLEARANCE/42X,2HIN)
WRITE(6,130)PR(1),PR(2),PRR,PD

```

```

160 READ(5,20)RPM(1),RPM(2),XF(2),XF(1),DFL11(2),DFL11(3),DFL11(1),
    DFL11(4),DFL11(5),FREE(2),FREE(1),FREE(1)
    IF(ABS(RPM(1))+ABS(RPM(2)).EQ.0.160 TO 10
    IF(ISTOP.EQ.0)GO TO 160
    ILOAD=ILOAD+1
    IQUIT=0
    TEMP1=.005*D/SRTA
    TEMP2=.005*D/CRTA
    DFL(1)=0.
    DFL(2)=0.
    DFL(3)=0.
    IF(XF(1).NE.0.)DFL(1)=-SIGN(TEMP1,XF(1))
    IF(XF(2).NE.0.)DFL(2)=-SIGN(TEMP2,XF(2))
    OME=.5*(RPM(1)*(1.+GAMM)+RPM(2)*(1.-GAMM))
    OMR=.5*(RPM(1)-RPM(2))*(1.-GAMM+2)/GAM
    FC=XMASS*.5*(E+2.*XBAR*SRMT02)*(.1047199*OME)**2
    G=XING*OME*OMR*SRMT02*1.09662E-2
    CALL OUTCON
    IF(IQUIT.EQ.1)GO TO 160
    WRITE(6,165)ILOAD,IBR
165 FORMAT(26H1 INPUT DATA FOR LOAD NO.,13,12H READING NO.,13)
    WRITE(6,170)
170 FORMAT(132H0 RPM OF LOADS APPLIED TO INNER IN
    INITIAL DISPLACEMENTS OF INNER WITH RESPECT TO OUTER ORBITAL
    2 ROTATIONAL/131H OUTER INNER ALONG Y ALONG Z
    3 ALONG X ALONG Y ALONG Z ABOUT X ABOUT Y VELOC
    4ITY VELOCITY/30X,9RHL9 LR IN RPM)
    5 IN RADIANS RADIANS RPM
    WRITE(6,130)RPM(1),RPM(2),XF(2),XF(1),DFL11(2),DFL11(3),DFL11(1),
    DFL11(4),DFL11(5),OME,OMR
    WRITE(6,180)GM,FC,FREE(2),FREE(1),FREE(1)
180 FORMAT(55H0 GYRO CENTRIFUGAL 1. = DEFLECTION PERMITTED/
    15AH MOVENT FORCE ALONG X ALONG Y ALONG Z/20H
    2 LB*IN LB/1P5E12.4)
    KKK=0
    DO 190 K=1,3
    IF(FREE(K).GT.0.)KKK=KKK+K**2
190 CONTINUE
    IF(KKK.EQ.0)GO TO 330

```

```

IF(FREE(1).EQ.0.)GO TO 330
SAV1=DFL(1)
SAV2=DFL(2)
SAV3=DFL(3)
DFL(1)=0.
IF(KKK.NE.1)GO TO 210
IF(FC*SRTA/CBTA.LT.-XF(1)/XN)GO TO 320
WRITE(6,200)
200 FORMAT(A24U) EXTERNAL THRUST IS NOT SUFFICIENT TO BALANCE INDUCED
1 THRUST - PROBLEM ABANDONED)
GO TO 160
210 DO 290 ITX=1,20
NLOAD=0
DO 220 K=1,5
XF1(K)=0.
DO 220 L=1,5
DTV(L,K)=0.
DO 250 JE=1,N
YI=J
JPASS=J
PHI=6.283185*(XJ-1.)/XN
SPH=SIW(PHI)
CPH=COS(PHI)
CALL ROLADU
IF(IQUIT)250,250,230
230 WRITE(6,240)ITX,J,(DFL(K),K=1,3)
240 FORMAT(19HU) VAIN PROGRAM 240,216,1P3F12.4)
GO TO 160
250 CONTINUE
IF(KKK.GT.5)GO TO 260
F22=XF1(2)+XF(2)
COR2=ER2/DTV(2,2)
DFL(2)=DFL(2)-COR2
IF(ABS(COR2)-TOL(4))310,200,290
260 IF(KKK.GT.10)GO TO 270
COR3=XF1(3)/DTV(3,3)
DFL(3)=DFL(3)-COR3
IF(ABS(COR3)-TOL(5))310,200,290
270 F22=XF1(2)+XF(2)

```

```

      X=XF1(3)
      DT=DTV(2,2)*DTV(3,3)-DTV(3,2)*DTV(2,3)
      COR2=(ER2+DTV(3,3)-ER3+DTV(2,3))/DET
      COR3=(DTV(2,2)*ER3-DTV(3,2)*ER2)/DET
      OFL(2)=OFL(2)-COR2
      OFL(3)=OFL(3)-COR3
      IF(ABS(COR2)-TOL(4))280,290,290
      IF(ABS(COR3)-TOL(5))310,290,290
280  CONTINUE
290  WRITE(6,300)KKK,(OFL(K),K=1,3)
300  FORMAT(19H VAIN PROGRAM 300,I6,IP3F12.4)
      GO TO 160
310  IF(XF1(1).LT.-XF(1))GO TO 320
      WRITE(6,200)
      GO TO 160
320  OFL(1)=SAV1
      OFL(2)=SAV2
      OFL(3)=SAV3
330  DO 530 ITER=1,20
      VLOADEN
      DO 340 K=1,5
      XF1(K)=0.
      DO 340 L=1,5
      DTV(L,K)=0.
340  DO 370 J=1,N
      JPASS=J
      XI=J
      P/I=6.293185*(XJ-1.)/XN
      SPH=SIGN(PHI)
      CPH=COS(PHI)
      CALL ROLoad
      IF(IQUIT)170,170,150
350  WRITE(6,360)ITER,KKK,J,(OFL(K),K=1,3)
360  FORMAT(19H VAIN PROGRAM 360,I6,IP3F12.4)
      GO TO 160
370  CONTINUE
      IF(KKK.EQ.0)GO TO 550
      IF(KKK.GT.1)GO TO 380
      COR1=(XF1(1)+XF(1))/DTV(1,1)

```

```

380 DFL(1)=DFL(1)-COR1
    IF(ABS(COR1)-TOL(3))550,530,530
    IF(KKK.GT.4)GO TO 390
    COR2=(XF1(2)+XF(2))/DTV(2,2)
    DFL(2)=DFL(2)-COR2
    IF(ABS(COR2)-TOL(4))550,530,530
    IF(KKK.GT.5)GO TO 410
    ER1=XF1(1)+XF(1)
    ER2=XF1(2)+XF(2)
    DET=DTV(1,1)*DTV(2,2)-DTV(2,1)*DTV(1,2)
    COR1=(ER1*DTV(2,2)-ER2*DTV(1,2))/DET
    COR2=(DTV(1,1)*ER2-DTV(2,1)*ER1)/DET
    DFL(1)=DFL(1)-COR1
    DFL(2)=DFL(2)-COR2
    IF(ABS(COR1)-TOL(3))400,530,530
    IF(ABS(COR2)-TOL(4))550,530,530
    IF(KKK.GT.9)GO TO 420
    COR3=XF1(3)/DTV(3,3)
    DFL(3)=DFL(3)-COR3
    IF(ABS(COR3)-TOL(5))550,530,530
    IF(KKK.GT.10)GO TO 440
    ER1=XF1(1)+XF(1)
    ER3=XF1(3)
    DET=DTV(1,1)*DTV(3,3)-DTV(3,1)*DTV(1,3)
    COR1=(ER1*DTV(3,3)-ER2*DTV(1,3))/DET
    COR3=(DTV(1,1)*ER3-DTV(3,1)*ER1)/DET
    IF(ABS(COR1)-TOL(3))430,530,530
    IF(ABS(COR3)-TOL(5))550,530,530
    IF(KKK.GT.13)GO TO 460
    ER2=XF1(2)+XF(2)
    ER3=XF1(3)
    DET=DTV(2,2)*DTV(3,3)-DTV(3,2)*DTV(2,3)
    COR2=(ER2*DTV(3,3)-ER3*DTV(2,3))/DET
    COR3=(DTV(2,2)*ER3-DTV(3,2)*ER2)/DET
    DFL(2)=DFL(2)-COR2
    DFL(3)=DFL(3)-COR3
    IF(ABS(COR2)-TOL(4))450,530,530
    IF(ABS(COR3)-TOL(5))550,530,530
    ERR(1)=XF1(1)+XF(1)
460 ERR(1)=XF1(1)+XF(1)

```

```

      FPR(2)=YF1(2)+XF(2)
      FPR(3)=YF1(3)
      KO=3
      DO 470 K=1,3
      DO 470 L=1,3
      AA(L,K)=DTV(L,K)
      CALL SIMULT(AA,KO,ERR,CORR,IQUIT)
      IF(IQUIT)500,500,480
      WRITE(6,490)ITER,KKK,(CORR(K),K=1,3),(DFL(K),K=1,3)
      FORMAT(19HU  WAIN PROGRAM 490,2I6,1P5F12.4)
      GO TO 160
470  DO 510 K=1,3
      DFL(K)=DFL(K)-CORR(K)
510  DO 520 K=1,3
      IF(ABS(CORR(K))-TOL(K+2))520,530,530
      CONTINUE
      GO TO 550
530  CONTINUE
      WRITE(6,540)KKK,(DFL(K),K=1,3)
540  FORMAT(19HO  WAIN PROGRAM 540,I6,1P3F12.4)
550  CALL OUTPT
      GO TO 160
      E'ID
      SURROUTINE ROLOAD
      COMMON ALPHA,AA(5,5),A(5,5),ABIG,  RFT1,R1,R2,RT1,BMTAU,MT02,
      1BT2AL,BGTMP,BR(2,60),  C(2),CROWN,CBTA,CTAU02,CBMTAU,CBMT02,
      2CBT2AL,CPH,CORR(5),CTH,COR(5),  D,NRNP,DELOI(5),DFL1(5),DFL(3)
      3,CTV(5,5),DELTA,DEL(2),DPDEL(2),DMDEL(2),CPH(2),CPH(2),DELX,
      4DTH,DA,DLX(2,60),DELSAV(2,60),DPID(5),DPID(5),DELL,DELLX,
      5E,EL(2),ERR(5),EFL,EX,ER(5),  FLI,FLT02,FREE(3),FC,  GAGE,GAM,
      6GAMM,GX,GW,  H1,H2,HERTZ(2,60),HR7,HQ,  IBR,ISTOP,ILOAD,IQUIT,
      7ICT,JPASS,  KKK,  N,NLOAD,  OME,QVR,  P,PR(2),PRR,PX(2),
      8P(2,60),P3,PX3(60),PIOUT,P3OUT,  R5,RHO,RV4,P1,R2,RP(2)
      9COMMON  S3TA,STAU02,SLANT,S8MTAU,SBT2AL,SAV1,SAV2,SAV3,SPH,STH,
      1  THD1(5),TAU02,TAU,TTAU02,TRT2AL,TOL(5),THETA,TH(2),TANTH,
      2THSAV(2,60),THET,TMPBIG,  V,VV,  WEIGHT,  XN,XLT,XLE,XIAU,
      3XLT02,XLE02,XNAR,XMASS1,XMASS2,XMASS,XINC61,XINC62,XINCG,YBAR1,
      4XRAR2,XRAR,XLEVER,XALPHA,XF(2),YF1(5),XV(2),XTH,XVEL,Y1,X',
      5X''M(2,60),XX(2,2,60),XHSAV(2,60),XMIOUT,YX0(2),YHBIG,  YM(2),

```

6Y'R
 DOUBLE PRECISION BGTMP,CTH,DELTA,DEL,DDEL,DMDEL,DPTH,DMTH,GMX,HD
 1,PX,P3,STH,TH,TANTH,VV,XM,XTH,XDEL
 DOUBLE PRECISION AID,A2D,A3D,A4D,BTAD,BMT2D,BMTAUD,BT2ALD,CRN,
 1CTAU2D,CRTAD,CMT2D,CRMTUD,CRT2AD,DD,NELXC,DNTHN,NSORT,DSIN,DCOS,
 2OYD,DLUG,DET,ED,ELD(2),EFLD,EXD,FLT02D,FCD,GMJ,PS1,PS2,P3DEL,P3TH
 3,PS1DEL,PS1TH,PS2DEL,PS2TH,P3DEL1,P3TH1,P3DEL2,P3TH2,PDEL,STAU2D,
 4SRTAD,SMT2D,SBMTUD,SRT2AD,SMINCD,SMD,TAU02D,TOLD(2),TEMPD,TEMPID,
 5XPRD,XLEVRD,XLE02D,XINCD,XHD,X1D,X2D,Y1D,Y2D
 DIMENSION PXS(2),XVS(2),NPDELS(2),DMDELS(2),DPHS(2),DMTHS(2)
 IF(1CT.GT.U)GO TO 5
 ICT=1
 BTAD=RTA
 BMT2D=BMT02
 BMTAUD=BMTAU
 RT2ALD=RT2AL
 CRN=CROWN
 CTAU2D=DCOS(TAU02D)
 CRTAD=CRTA
 CMT2D=DCOS(BMT2D)
 CRMTUD=DCOS(BMTAUD)
 CRT2AD=DCOS(BT2ALD)
 D7=D
 E7=E
 EL0(1)=EL(1)
 EL0(2)=EL(2)
 FLT02D=FLT02
 STAU2D=DSIN(TAU02D)
 SRTAD=DSIN(BTAD)
 SMT2D=DSIN(BMT2D)
 SMTUD=DSIN(BMTAUD)
 SRT2AD=DSIN(BT2ALD)
 TAU02D=TAU02
 TOLD(1)=TOLD(1)
 TOLD(2)=TOLD(2)
 XPRD=XPR
 XLEVRD=XLEVER
 XLE02D=XLE02
 J=JPASS

```

F00=FC
S00=GM
VV=COS(ALPHA-TAU02)/COS(ALPHA+TAU02)
DELTA=(DEL(1)+DEL11(1))*SRTA+.5*(E*SRTA+D*STAU02)*(DFL11(4)*SPH+
1DFL11(5)*CPH)+(DEL(2)+DEL11(2))*CPH+(DEL(3)+DEL11(3))*SPH-.5*P7)
2*CRTA
DEL(1)=.5500*DELTA
DEL(2)=(DELTA-DEL(1))*VV
THETA=JFL11(4)*SPH+DFL11(5)*CPH
T1(1)=.500*THETA
T4(2)=THETA-T1(1)
DO 140 J=1,20
DO 120 K=1,2
PV(K)=0.00
XV(K)=0.00
DPDEL(K)=0.00
DDEL(K)=0.00
DPTH(K)=0.00
DPTH(K)=0.00
XTH=TH(K)
CTH=DCOS(XTH)
STH=DSIN(XTH)
TANTH=STH/CTH
XDEL=DEL(K)
C'LL XIREME(CRM,CTH,FLI02D,HD,KV,STH,X1,X2D,XDFL,XLE02D)
I'(KM)20,20,10
P(1,J)=PIOUT
X'M(1,J)=XM1OUT
PX(1)=PIOUT
X'(1)=XM1OUT
P(2,J)=0.
PYS(1)=PIOUT
X'S(1)=XM1OUT
NOLOAD=NOLOAD+1
GO TO 270
EFLD=X10-X2D
XTNCD=EFL7/30.00
XXY(1,K,J)=X10
XXY(2,K,J)=X20

```

10

20

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XHD=X1D+XINCD
SMINCD=1.00
RGTMP=C.00
DO 110 L=1,31
SMD=3.00-SMINCD
SMINCD=-SMINCD
IF((L.EQ.1).OR.(L.EQ.31))SMD=1.00
XHD=XHD-XINCD
DELXD=XDEL+XHD*TANTH
DOTH=XHD/CTH**2
IF(DABS(XHD).LE.FLT02D*CTH)GO TO 60
TEMPD=XHD-HD*STH
TEMPID=CRN**2-TEMPD**2
IF(TEMPID)30,30,50
IF(TEMPID)30,30,50
IAUIT=1
WRITE(6,40)I8R,ILOAD,IT,J,L,K,(DFL(M),M=1,3)
FORMAT(12H0 ROLOAD 40,6I6,1P3E12.4)
RETURN
30 TEMPID=DSORT(TEMPID)
DELXD=TEMPID-HD*CTH+XDEL
DOTH=TEMPD+HD*CTH/TEMPID+HD*STH
IF(DELXD.LT.1.0-8)GO TO 110
DXD=DD+2.00*XHD*STAU2D/CTAU2D
EXD=ED+2.00*XHD*SRTAD+DD*CBMT2D-DXD*CRTAD
GX=XDXD*CRTAD/EXD
IF(K.EQ.2)GMX=-GXN*CBMTUN/EXD
TEMPD=D.07*DELXD**1.11111/EFLO**1.11111
DO 70 ITF=1,20
IF(TEMPD.LE.0.00)GO TO 110
AID=ELD(K)*OXD*TEMPD*(1.00+GMX)
AID=DSORT(AID)
A2D=1.8864D0+DLOG(EFLO*.5D0/AID)
A3D=ELD(K)*A2D*TEMPD
A4D=(A3D-DELXD)/((A2D-.500)*ELD(K))
TEMPD=TEMPD-A4D
IF(DABS(A3D-DELXD)-TOLD(1))90,70,70
CONTINUE
70
WRITE(6,80)I8R,ILOAD,IT,J,K,A4D,(DFL(M),M=1,3)
FORMAT(12H0 ROLOAD 80,5I6,1P4E12.4)
80

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```

          IQUIT=1
          RETURN
90      PY(K)=PX(K)+TEMPD*SMO
          XM(K)=XM(K)+XHD*TEMPD*SMO
          PDEL=SMO/((A2D-.5D)*FLN(K))
          OPDEL(K)=OPDEL(K)+PDEL
          OPTH(K)=OPTH(K)+DOTH*PDEL
          OMDL(K)=OMDL(K)+XHD*PDEL
          OMTH(K)=OMTH(K)+XHD*OMTH*PDEL
          IF(TEMPD-AGTMP)110,110,100
          AGTMP=TEMPD
100      DLX(K,J)=DELX
          HERTZ(K,J)=.6366198D0*TEMPD/AID
          BR(K,J)=2.D0*AID
          XMSAV(K,J)=XHD
110      CONTINUE
          IF(PX(K).EQ.D.0)60 TO 10
          TEMPD=XINC/3.00
          PX(K)=PX(K)+TEMPD
          XM(K)=XM(K)+TEMPD
          OPDEL(K)=OPDEL(K)+TEMPD
          OPTH(K)=OPTH(K)+TEMPD
          OMDL(K)=OMDL(K)+TEMPD
          OMTH(K)=OMTH(K)+TEMPD
120      CONTINUE
          P3=(-XM(1)+XM(2)-GMD+PCD+XBARD*CAWT2D-(PX(1)-PX(2))*5D0*RD*ST.U2D
            1)/XLEVRD
          PS1=-PX(1)*CBTAD+PX(2)*CAWTUD+PCD-P3*CBT2AD
          PS2=-PX(1)*SBTAD+PX(2)*SBWTUD+P3*CBT2AD
          PDEL=(-(OPDEL(1)+OPDEL(2)*VV)*5D0*RD*STAU2D-OMDEL(1)-OMDEL(2)*VV
            1)/XLEVRD
          P3TH=(-(OPTH(1)+OPTH(2))*5D0*RD*STAU2D-OMTH(1)-OMTH(2))/YLEVRD
          PS1DEL=-OPDEL(1)*CBTAD-OPDEL(2)*VV*CAWTUD-P3DEL*CBT2AD
          PS1TH=-OPTH(1)*CBTAD-OPTH(2)*CBWTUD-P3TH*CBT2AD
          PS2DEL=-OPDEL(1)*SBTAD-OPDEL(2)*VV*SBWTUD+P3DEL*CBT2AD
          PS2TH=-OPTH(1)*SBTAD-OPTH(2)*SBWTUD+P3TH*CBT2AD
          OFTD=PS1DEL*PS2TH-PS2DEL*PS1TH
          Y1D=(PS1*PS2TH-PS2*PS1TH)/DETD
          Y2D=(PS1DEL*PS2-PS2DEL*PS1)/DETD

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130 DFL(1)=DEL(1)-Y1D
140 DFL(2)=(DELTA-DEL(1))*VV
    TH(1)=TH(1)-Y2D
    TH(2)=THETA-TH(1)
    IF(DARS(Y1D)-TOLD(1))130,140,140
    IF(DARS(Y2D)-TOLD(2))160,140,140
140 CONTINUE
150 WRITE(6,150)IRR,ILOAD,J,Y1D,Y2D,(DFL(M),M=1,3)
    FORMAT(13H0 ROLOAD 150,3I6,1P5=12.4)
    IQUIT=1
    RETURN
160 DO 170 K=1,2
    P(K,J)=PX(K)
    XM(K,J)=XM(K)
    DFLSAV(K,J)=DFL(K)
    THSAV(K,J)=TH(K)
    PX3(J)=P3
170 P3DEL1=(-OMDEL(1)-DPDEL(1)*.5D0*DD*STAU2D)/XLEVRO
    P3TH1=(-OMTH(1)-DPTH(1)*.5D0*DD*STAU2D)/XLEVRO
    P3DEL2=(OMDEL(1)+DPDEL(2)*.5D0*DD*STAU2D)/XLEVRO
    P3TH2=(OMTH(2)+DPTH(2)*.5D0*DD*STAU2D)/XLEVRO
    A(1,1)=-DPDEL(1)*CBTAD-P3DEL1*SAT2AD
    A(1,2)=DPDEL(2)*CBMTUD-P3DEL2*SAT2AD
    A(1,3)=-DPH(1)*CRTAD-P3TH1*SAT2AD
    A(1,4)=DPTH(2)*CBMTUD-P3TH2*SAT2AD
    A(2,1)=-DPDEL(1)*SBTAD+P3DEL1*CAT2AD
    A(2,2)=DPDEL(2)*SBMTUD+P3DEL2*CAT2AD
    A(2,3)=-DPTH(1)*SATAD+P3TH1*CAT2AD
    A(2,4)=DPTH(2)*SBMTUD+P3TH2*CAT2AD
    A(3,1)=1.
    A(3,2)=1.7U/VV
    A(3,3)=0.
    A(3,4)=0.
    A(4,1)=0.
    A(4,2)=0.
    A(4,3)=1.
    A(4,4)=1.
    ER(1)=0.
    EP(2)=0.

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      F(3)=1.
      F(4)=0.
      DO 180 K=1,4
      180 L=1,4
      A(L,K)=A(L,K)
      M=4
      M=1
      CALL S.MULT(A,N7,ER,COR,IQUIT)
      IF(IQUIT)20,220,200
      200 WRITE(6,210) IPR,ILOAD,J,(OFL(K),K=1,3)
      210 FORMAT(1340  PLOAD 210,3I6,1P3F12.4)
      RETURN
      GO TO(230,250),MM
      230 M=2
      DO 240 K=1,4
      240 L=1,4
      A(L,K)=AA(L,K)
      OFLDEL=COR(1)
      THDEL=COR(3)
      F(3)=0.
      F(4)=1.
      GO TO 180
      OFLTH=COR(1)
      THTH=COR(3)
      OFLD1(2)=CBTA*CPH
      OFLD1(3)=CBTA*SPH
      OFLD1(4)=.5*(E*SBTA+D*STAU02)*SPH
      OFLD1(5)=.5*(F*SBTA+D*STAU02)*CPH
      THD1(4)=SPH
      THD1(5)=CPH
      DO 255 K=1,2
      255 L=1,2
      DPDEL(K)=JPDFL(K)
      DPDEL(K)=JMDFL(K)
      DPTHS(K)=JPTH(K)
      DWMTHS(K)=JWMTH(K)
      PYS(K)=PX(K)
      XYS(K)=XY(K)
      CONTINUE
      GO 260 L=1,5

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DPID1(L)=(DPDELS(1)*DELDEL+DPTHS(1)*THDEL)*DELJ1(L)+(OPDELS(1)*
10FLTH+DPTHS(1)*THTH)*THJ1(L)
260 DM101(L)=(DMDELS(1)*DELDEL+DMTHS(1)*THDEL)*DELJ1(L)+(DMDELS(1)*
10FLTH+DMTHS(1)*THTH)*THJ1(L)
270 XF1(1)=XF1(1)+PXS(1)*SRTA
XF1(2)=XF1(2)+PXS(1)*CBTA*CPH
XF1(3)=XF1(3)+PXS(1)*CBTA*SPH
TEMP=.5*(E*SRTA+D*STAU02)*PXS(1)+XMS(1)
XF1(4)=XF1(4)+TEMP*SPH
XF1(5)=XF1(5)+TEMP*CPH
IF(PXS(1).EQ.0.)RETURN
DO 280 L=1,5
DTV(1,L)=DTV(1,L)+DPID1(L)*SRTA
DTV(2,L)=DTV(2,L)+DPID1(L)*CBTA*CPH
DTV(3,L)=DTV(3,L)+DPID1(L)*CBTA*SPH
TEMP=.5*(E*SRTA+D*STAU02)*DPID1(L)+DM101(L)
DTV(4,L)=DTV(4,L)+TEMP*SPH
DTV(5,L)=DTV(5,L)+TEMP*CPH
RETURN
END
SURROUTINE OUTCON
COMMON ALPHA,AA(5,5),A(5,5),ABIG, BFIA,P1,R2,RTA,BMTAU,BMT02,
1BT2AL,BGTP,BB(2,60), C(2),CROWN,CBTA,CTAU02,CBMTAU,CBMT02,
2CBT2AL,CPH,CORR(5),CTH,COR(5), D,DROP,DELD1(5),DFL1(5),DFL(3)
3,DTV(5,5),DELTA,DEL(2),DPDEL(2),DMDEL(2),DPTH(2),DMTH(2),DELX,
4DNTH,DX,DLX(2,60),DELSAV(2,60),DM101(5),DM101(5),DELL,DELLX,
5E,EL(2),ERR(5),EFL,EXER(5), FLT,FLT02,FREE(3),FC, GAGE,GAM,
6GAMM,GM,GMX, H1,H2,HERIZ(2,60),HR7,HD, IBR,ISTOP,ILOAD,IQUIT,
7ICT,JPASS, KKK, N,NOLOAD, OME,OMR, P,PR(2),PRR,PX(2),
8P(2,60),P3,PX3(60),P1OUT,P3OUT, RS,PHO,RWH,P1,R2,RPW(2)
COMMON SRTA,STAU02,SLANT,SBMTAU,S3T2AL,SAV1,SAV2,SAV3,SPH,STH,
1 THD1(5),TAU02,TAU,TTAU02,TRT2AL,TOL(5),THETA,TH(2),TANTH,
2THSAV(2,60),THET,TMPBIG, V,VV, WEIGHT, XN,XLT,VLE,XTAU,
3XLT02,XLE02,XNAR,XMASS1,XMASS2,XMCG1,XMCG2,XMCG,XBAR1,
4XRAR2,XRAR,XLEVER,XALPHA,XF(2),XF1(5),XV(2),XTH,XDEL,X1,X2,
5XVM(2,60),XX(2,2,60),XHS(2,60),XHS(2,60),XHS(2,60),XHS(2,60),XHS(2,60),
6Y,YR
DOUBLE PRECISION BGTP,CTH,DELTA,DEL,DPDEL,DMDEL,DPTH,DMTH,GMX,HD
1,PX,P3,STH,THETA,TH,TANTH,VV,XM,XTH,XDEL

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DOUBLE PRECISION C,N,CNTHD,DLD,FLT0,SNTHN,XL0,X1,XD2
NLL=.UN5*U
THET=0.
DO 100 ITR=1,25
  PIOUT=0.
  X1OUT=0.
  PIDEL=0.
  PITH=0.
  X1DEL=0.
  X1TH=0.
  SNTH= SIN(THET)
  CNTH= COS(THET)
  C=N*CRUWN
  CNTHD=CNTH
  DLD=DELL
  FLT0=FLT02
  SNTHD=SNTH
  XLD=XLE02
  CALL XREME(C,N,CNTHD,FLT0,H0,K4,SNTHN,Y01,X02,PL0,XL0)
  X1=XD1
  X2=XD2
  X0(1)=Y1
  X0(2)=X2
  EFL=X1-X2
  SMINC=1.
  XINC=EFL/30.
  XH=X1+XINC
  TPBIG=0.
  DO 80 L=1,31
    XH=XH-XINC
    DY=D+2.*X4*TTAU02
    EX=E+2.*XH*SRTA+D*CBMT02-DX*CBTA
    GAM2=DX*CBTA/EX
    SM=3.-SMINC
    SMINC=-SMINC
    IF((L.EQ.1).OR.(L.EQ.31))SM=1.
    DELX=DELL+XH*SNTH/CNTH
    D0TH=XH/CNTH**2
    IF(ABS(XH).LE.FLT02*CNTH)GO TO 40

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10  TEMP=XH-RMH*SNTH
    TEMP1=CROWN**2-TEMP**2
    IF (TEMP1) 10,10,30
    ICUTE=1
    WRITE(6,20) IBR, ILOAD, IIR, L, DELL, THET
20  FORMAT(12H0 OUTCON 20,4I6,1P2E12.4)
    RETURN
30  TEMP1=SQRT(TEMP1)
    DELX=TEMP1-RMH*CNTH+DELL
    DOTH=TEMP+RMH*CNTH/TEMP1+RMH*SNTH
    IF (DELX, LT, 1.E-8) GO TO 40
    TEMP=5.E7*JELY**1.111111/EFL**1.111111
    DO 50 ITQ=1,20
    IF (TEMP, LE, 0.) GO TO 80
    A1=EL(1)*CX*TEMP*(1.+GAMP)
    A1=SQRT(A1)
    A2=1.A964+ALOG(EFL*.5/A1)
    A3=EL(1)*A2*TEMP
    A4=(A3-DELX)/((A2-.5)*EL(1))
    TEMP=TEMP-A4
    IF (ABS(A3-DELX)-TOL(1)) 70,50,50
50  CONTINUE
    WRITE(6,60) IBR, ILOAD, IIR, A4, DELL, THET
60  FORMAT(12H0 OUTCON 60,3I6,1P3E12.4)
    ICUTE=1
    RETURN
70  PIOUT=PIOUT+TEMP*SM
    X'1OUT=X'1OUT+XH*TEMP*SM
    PDEL=SM/((A2-.5)*EL(1))
    PIDEL=P1DEL+PDEL
    X'1DEL=X'1DEL+XH*PDEL
    P1TH=P1TH+DDTH*PDEL
    X'1TH=X'1TH+XH*DDTH*PDEL
    IF (TEMP, LE, TMRIG) GO TO 40
    TMRIG=TEMP
    ARIG=2.*A1
    DELLX=DELL
    X'1RIG=XH
    HRZ=TEMP/ARIG

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30  CONTINUE
    TEMP=XINC/3.
    P1OUT=P1OUT+TEMP
    X1OUT=X1OUT+TEMP
    P1DEL=P1DEL+TEMP
    X1DEL=X1DEL+TEMP
    P1TH=P1TH+TEMP
    X1TH=X1TH+TEMP
    P1OUT=(-P1OUT+.5*D*STAU02+FC*XBAR*CRMT02-6M-XM1OUT)/XLEVER
    P1DEL=(-P1DEL+FC-P3OUT*SBT2AL
    P2=-P1OUT*SBTA+P3OUT*CR2AL
    P1DEL=(-.5*D*STAU02*P1DEL-XM1DEL)/XLEVER
    P1TH=(-.5*D*STAU02*P1TH-XM1TH)/XLEVER
    P1DEL=-CR1A*P1DEL-P3DEL*SBT2AL
    P1TH=-CR1A*P1TH-P3TH*SBT2AL
    P2DEL=-SB1A*P1DEL+P3DEL*CR2AL
    P2TH=-SB1A*P1TH+P3TH*CR2AL
    DFT=PS1DEL*PS2TH-PS2DEL*PS1TH
    COR1=(PS1*PS2TH-PS2*PS1TH)/DFT
    COR2=(PS1DEL*PS2-PS2DEL*PS1)/DET
    NFLL=DELL-COR1
    THET=THET-COR2
    IF (ABS(COR1))-TOL(1)) 90,100,100
    IF (ABS(COR2))-TOL(2)) 120,100,100
100  CONTINUE
    WRITE(6,110) IPR,ILOAD,ITR,COR1,COR2,DELL,THET
110  FORMAT(13HU OUTCON 11U,316,1P4F12.4)
    TQUIT=1
120  RETURN
    END
    SUBROUTINE OUTPT
    COMMON ALPHA,AA(5,5),A(5,5),ABIG, RFT1,R1,R2,RTA,BMTAU,BMT02,
    IRT2AL,BCTMP,RR(2,60), C(2),CROWN,CBTA,CTAU02,CBMTAU,CBMT02,
    2CBT2AL,CPH,CORR(5),CTH,COR(5), D,CROWN,DELO1(5),DFL11(5),DFL(3)
    3,CTV(5,5),DELTA,DEL(2),DPDEL(2),DMDEL(2),DPH(2),DMTH(2),DELX,
    4CTH,CA,DLX(2,60),DELSAV(2,60),CPID1(5),CPID1(5),DELL,DELLX,
    5F,EL(2),ERR(5),FFL,EX,ER(5), FLT,FLT02,FREE(3),FC, SAGE,GAM,
    6GMM,GM,GMA, H1,H2,HERIZ(2,60),HR7,H0, IBR,ISTOP,ILOAD,IQUIT,
    7ICT,JPASS, KKK, N,NLOAD, OME,OMR, PO,PR(2),PRR,PX(2),

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8P(2,60),P3,PX3(60),P10UT,P3OUT, RS,PHO,R4H,P1,R2,RP"(2)
COMMON SRTA,STAU02,SLANT,SBMTAU,SBT2AL,SAV1,SAV2,SAV3,SP4,STH,
1 THD1(5),TAU02,TAU,TTAU02,TRT2AL,TOL(5),THETA,TH(2),TANTH,
2THSAV(2,60),THET,TMPBIG, V,VV, WEIGHT, XN,XLT,YLE,XTAU,
3XLT02,XLE02,XNAB,XMASS1,XMASS2,XMASS,XINCG1,XINCG2,XINCG,YBAR1,
4XRAR2,XPAR,XLEVER,XALPHA,XF(2),XF1(5),XV(2),XTH,XDEL,Y1,X2,
5XVM(2,60),XXX(2,2,60),X4SAV(2,60),XW1OUT,XX0(2),XHBIG, YV(2),
6Y"R
DOUBLE PRECISION RGMP,CTH,DELTA,DEL,DDEL,DMDEL,DPTH,DMTH,GMX,HD
1,PX,P3,STH,THETA,TH,TANTH,VV,XM,XTH,XDEL
WRITE(6,10)ILOAD,IBR
10 FORMAT(10 OUTPUT DATA FOR LOAD NO.,I3, BEARING NO.,I3)
WRITE(6,20)
20 FORMAT(10,15X, REACTIONS OF BEARING ON SHAFT,22X, TOTAL DISPLACE
1MENTS OF INNER WITH RESPECT TO OUTER,10 ALONG X ALONG Y
2ALONG Z ABOUT X ABOUT Y ABOUT X ALONG X ALONG Y ALONG
3Z ABOUT X ABOUT Y,6X,LR,10X,LR,10X,LR,10X,LR,AX,LB*IN,7X
4, LB*IN,AX,IN,10X,IN,10X,IN,AX, RADIANS,5X, RADIANS,1)
DEL(1)=DFL(1)+DFL11(1)
DEL(2)=DFL(2)+DFL11(2)
DEL(3)=DFL(3)+DFL11(3)
WRITE(6,30)XF1(2),XF1(3),XF1(4),XF1(5),DFL(2),DFL(3),DFL(1
1),DFL11(4),DFL11(5)
30 FORMAT(1P11E12.4)
WRITE(6,40)
40 FORMAT(10 ROLL ROLL CONTACT LOAD CONT
1ACT MOMENT OUTER PATH EXTREMITY INNER PATH EXTREMITY
2FLANGE,10 NUMBER X(1) X(2) X(1) X(2)
3 INNER X(1) X(2) X(1) X(2)
4 LOAD,17X,NEG,10X,LR,10X,LR,AX,LR*IN,7X,LB*IN,9X,IN,
510X,IN,10X,IN,10X,IN,10X,LR,
DO 70 J=1,N
XJ=J
PUT=360.*(XJ-1.)/YN
IF(P(2,J).EQ.0.)GO TO 70
WRITE(6,60)J,PHI,P(1,J),P(2,J),XMM(1,J),YMM(2,J),XXX(1,1,J),XXX(2
1,1,J),XXX(1,2,J),XXX(2,2,J),PX3(J)
60 FORMAT(17,5X,1P10F12.4)
70 CONTINUE

```

```

IF(NOLoad.EQ.0)GO TO 90
WRITE(6,80)PIOUT,X*10UT,XX0(1),XX0(2),PZ*UT
FORMAT(10 OTHERS,14X,1PIE12.4,12X,1PIF12.4,12Y,1P212.4,24X,1P1
1F12.4)
90 WRITE(6,100)
100 FORMAT(10 ROLL,9X,1CONTACT LENGTH,4X,1MAXIMUM CONTACT WIDTH
1MAX. CONTACT DEFLECTION 1MAXIMUM HERTZ STRESS LOCATION OF MAX. V
2ALUES,10 NUMBER,6X,1OUTER,7X,1INNER,7X,1OUTER,7X,1INNER,7X
3,1OUTER,7X,1INNER,7X,1OUTER,7X,1INNER,7X,1OUTER,7X,1INNER,19
4X,1IN,10X,1IN,10X,1IN,10X,1IN,10X,1IN,10X,1IN,7X,1L/IN*2,
54Y,1L/IN*2,7X,1IN,10X,1IN)
DO 110 J=1,N
IF(P(2,J).EQ.0)GO TO 110
TEMP=XXX(1,1,J)-XXX(2,1,J)
TEMP1=XXX(1,2,J)-XXX(2,2,J)
WRITE(6,60)J,TEMP,TEMP1,RR(1,J),RB(2,J),RLX(1,J),RLX(2,J),HERTZ(1,
1 J),HERTZ(2,J),XHSAB(1,J),XHSAB(2,J)
110 CONTINUE
IF(NOLoad)135,135,120
120 CONL=XX0(1)-XX0(2)
WRITE(6,130)CONL,ABIG,DELLX,HRZ,XHBIG
FORMAT(10 OTHERS,1PIE14.4,12X,1PIE12.4,12Y,1PIF12.4,12Y,1PIE12.
14,12X,1PIF12.4)
135 WRITE(6,140)
140 FORMAT(10 ROLL CONTACT MISALIGNMENT,10 NUMBER OUTER
1 INNER,15X,1RADIANS 1RADIANS,1)
DO 150 J=1,N
IF(P(2,J).EQ.0)GO TO 150
WRITE(6,60)J,THSAV(1,J),THSAV(2,J)
150 CONTINUE
IF(NOLoad.EQ.0)GO TO 165
ZTHETA=THET*57.29578
WRITE(6,160)ZTHETA
FORMAT(10 OTHERS 1,1PIF12.4)
160 WRITE(6,170)
165 WRITE(6,170)
170 FORMAT(10 PARTIAL DERIVATIVES OF REACTIONS WITH RESPECT TO DISPLA
1CMENTS,10 OFX/DX OFY/DY OFX/DZ OFY/DZ OFX/DZ OFY/DZ
2ALY,10 L/IN L/IN L/IN L/IN L/IN L/IN L/IN L/IN L/IN L/IN L/IN
WRITE(6,180)OTV(2,2),OTV(2,3),OTV(2,1),OTV(2,4),OTV(2,5)

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```

180 FORMAT(1P5E12.4)
WRITE(6,190)
190 FORMAT(10 DFY/DX LR/IN LB/IN DFY/DZ LB/RAD DFY/DAL LB/RAD)
WRITE(6,190)DTV(3,2),DTV(3,3),DTV(3,1),DTV(3,4),DTV(3,5)
WRITE(6,200)
200 FORMAT(10 DFZ/DX LR/IN LB/IN DFZ/DZ LB/RAD DFZ/DAL LB/RAD)
WRITE(6,200)DTV(1,2),DTV(1,3),DTV(1,1),DTV(1,4),DTV(1,5)
WRITE(6,210)
210 FORMAT(10 DMX/DX LBIN/IN LBIN/IN DMX/DZ LBIN/RAD LBIN/RAD)
WRITE(6,210)DTV(4,2),DTV(4,3),DTV(4,1),DTV(4,4),DTV(4,5)
WRITE(6,220)
220 FORMAT(10 DMY/DX LBIN/IN LBIN/IN DMY/DZ LBIN/RAD LBIN/RAD)
WRITE(6,220)DTV(5,2),DTV(5,3),DTV(5,1),DTV(5,4),DTV(5,5)
RETURN
END
SUBROUTINE RCALC(CROWN,DROP,FLT02,GAGF,HD,XLE02)
DOUBLE PRECISION CRD,DNP,DSORT,FLT2D,GAG,HD,XLE02D
FLT2D=FLT02
GAG=GAGF
XLE02D=XLE02
IF(CROWN)10,10,20
DNP=DROP
CRD=DSORT(((XLE02D-GAG)**2-FLT2D**2+DNP**2)/(2.0*DNP))**2+
1FLT2D**2)
HD=DSORT(CRD**2-FLT2D**2)
CROWN=CRD
RETURN
CRD=CROWN
HD=DSORT(CRD**2-FLT2D**2)
DNP=HD-DSORT(CRD**2-(XLE02D-GAG)**2)
RETURN
END
SUBROUTINE XTREME(CRN,CTH,FLT02D,HD,KW,STH,X1D,X2D,XDEL,XLE02D)
DOUBLE PRECISION CRN,CTH,DSORT,DNABLA,FLT02D,HD,STH,STHAB,TEVP,
1XSTAR1,XSTAR2,X1D,X2D,XDEL,XLE02D

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      STHAR=JARC(STH)
      V=0
      TEMP=C*H**2-(H*CTH-XDEL)**2
      IF(TEMP)10,10,20
10      K=1
      RETURN
20      TEMP=DSOPT(TEMP)
      V1D=TEMP+H*STHAB
      X2D=TEMP+H*STHAR
      D1ABLA=H*DSOPT(C*H**2-XLE02D**2)
      XSTAR1=XLE02D*CTH+D1ABLA*STHAR
      XSTAR2=-XLE02D*CTH+D1ABLA*STHAB
      IF(X2D.GT.XSTAR1)GO TO 10
      IF(X1D.LE.FLT02D*CTH)GO TO 10
      IF(X1D.GT.XSTAR1)X1D=XSTAR1
      IF(X2D.LT.XSTAR2)X2D=XSTAR2
      IF((X2D.GT.-FLT02D*CTH).AND.(X2D.LT.FLT02D*CTH))X2D=-XDEL*CTH/
15THAR
      IF(STH.GE.0.00)RETURN
      TEMP=-X1D
      X1D=-X2D
      X2D=TEMP
      RETURN
      END
      SUBROUTINE SIMULT(A,N,RQ,XX,KX)
      DIMENSION AA(5,5),BB(5),XX(5),KOL(5)
      DOUBLE PRECISION A(5,5),R(5),X(5),ROW(5),TEMP,AMPY
      DO 10 J=1,N
      R(J)=RQ(J)
      DO 10 J=1,N
      A(J,I)=AA(J,I)
      DO 505J J=1,N
      TEMP=0.00
      DO 505I I=1,N
      IF(DABS(A(J,K))-TEMP) 5051,5051,5052
5052 TEMP=DABS(A(J,K))
5051 CONTINUE
      DO 5053 K=1,N
5053 A(J,K)=A(J,K)/TEMP

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SIMU0050
 SIMU0060
 SIMU0070
 SIMU0080
 SIMU0090
 SIMU0100
 SIMU0110
 SIMU0120
 SIMU0130
 SIMU0140
 SIMU0150
 SIMU0160

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R(J)=R(J)/TEMP
5050 CONTINUE
5000 KOL(1)=1
5001 DO 5002 IROW=2,N
5002 KOL(IROW)=KOL(IROW-1)+1
5004 DO 5025 KOUNT=1,N
  LARGST=N-KOUNT+1
  IFRASE=KOL(1)
  JCOL=1
5005 IF (N-KOUNT) 5035,5014,5006
5006 AMPY=DABS ( A(1,1) )
5007 DO 5010 IROW=2,LARGST
5008 IF ( AMPY -DABS ( A(IROW,1))) 5009, 5010, 5010
5009 JCOL=IROW
  AMPY=DABS ( A(IROW,1) )
  IFRASE=KOL(IROW)
5010 CONTINUE
5011 IF (KOL(1)-IFRASE) 5012,5014,5012
5012 KOL(JCOL)=KOL(1)
  KOL(1)=IFRASE
5014 IF(A(JCOL,1))5015,5035,5015
5015 AMPY=A(JCOL,1)
5017 DO 5018 IROW=2,N
  ROW(IROW-1)=A(JCOL,IROW)/AMPY
5018 A(JCOL,IROW-1)=A(1,IROW-1)
  ROW(N)=1.00/AMPY
  A(JCOL,N)=A(1,N)
5019 DO 5022 IROW=2,N
  AMPY=A(IROW,1)
5020 DO 5021 JCOL=2,N
5021 A(IROW-1,JCOL-1)=A(IROW,JCOL)-AMPY*ROW(JCOL-1)
5022 A(IROW-1,N)=AMPY*ROW(N)
5023 DO 5024 JCOL=1,N
  KOL(JCOL)=KOL(JCOL+1)
5024 A(N,JCOL)=ROW(JCOL)
5025 KOL(N)=IFRASE
5026 DO 5034 KOUNT=1,N
5027 IF (KOL(KOUNT)-KOUNT) 5035,5034,5028
5028 DO 5032 IROW=KOUNT,N

```

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SIMU0170
SIMU0180
SIMU0190
SIMU0200
SIMU0210
SIMU0220
SIMU0230
SIMU0240
SIMU0250
SIMU0260
SIMU0270
SIMU0280
SIMU0290
SIMU0300
SIMU0310
SIMU0320
SIMU0330
SIMU0340
SIMU0350
SIMU0360
SIMU0370
SIMU0380
SIMU0390
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SIMU0490
SIMU0500
SIMU0510
SIMU0520
SIMU0530
SIMU0540
SIMU0550

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```

5029 IF (KOL(IROW)-KOUNT) 5035,5030,5032
5030 GO 5031 JC=1,N
      RW(1)=A(JCOL,IROW)
      A(JCOL,IROW)=A(JCOL,KOUNT)
5031 A(JCOL,KOUNT)=RW(1)
      IFRASE=KOL(KOUNT)
      KOL(KOUNT)=KOL(IROW)
      KOL(IROW)=IFRASE
      GO TO 5034
5032 CONTINUE
      GO TO 5035
5034 CONTINUE
997 IF(KX-3)998,9000,998
9000 KY=0
      RETURN
998 GO 5042 IROW=1,N
5040 X(IROW)=0,J0
5041 DO 5042 JKOL=1,N
5042 X(IROW)=X(IROW)+A(IROW,JKOL) * B(JKOL)
      KY=0
      GO 6000 IF=1,N
6000 XY(I)=X(I)
      RETURN
5035 KY=1
      RETURN
      END

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SI'U0567
 SI'U0570
 SI'U0577
 SI'U0590
 SI'U0607
 SI'U0610
 SI'U0627
 SI'U0630
 SI'U0640
 SI'U0650
 SI'U0660
 SI'U0670
 SI'U0680
 SI'U0690
 SI'U0700
 SI'U0710
 SI'U0720
 SI'U0730
 SI'U0740
 SI'U0750
 SI'U0760
 SI'U0777
 SI'U0780
 SI'U0790
 SI'U0800
 SI'U0810

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